

On the Linkage between Planck's Quantum and Maxwell's Equations

Tuomo Suntola

Physics Foundations Society, www.physicsfoundations.org

Abstract

The concept of quantum is one of the key elements of the foundations in modern physics. The term quantum is primarily identified as a discrete amount of something. In the early 20th century, the concept of quantum was needed to explain Max Planck's blackbody radiation law which suggested that the electromagnetic radiation emitted by atomic oscillators at the surfaces of a blackbody cavity appears as discrete packets with the energy proportional to the frequency of the radiation in the packet.

The derivation of Planck's equation from Maxwell's equations shows quantizing as a property of the emission/absorption process rather than an intrinsic property of radiation; the Planck constant becomes linked to primary electrical constants allowing a unified expression of the energies of mass objects and electromagnetic radiation thereby offering a novel insight into the wave nature of matter.

From discrete atoms to a quantum of radiation

Continuity or discontinuity of matter has been seen as a fundamental question since antiquity. Aristotle saw perfection in continuity and opposed Democritus's ideas of indivisible atoms. First evidences of atoms and molecules were obtained in chemistry as multiple proportions of elements in chemical reactions by the end of the 18th and in the early 19th centuries. For physicist, the idea of atoms emerged for more than half a century later, most concretely in statistical thermodynamics and kinetic gas theory that converted the mole-based considerations of chemists into molecule-based considerations based on conservation laws and probabilities.

A linkage between matter and radiation was first observed as heat radiation; any material body was observed to send radiation, visible light or infrared, depending on the temperature of the body. A closer study of the radiation spectrum showed that the wavelength distribution of the radiation is specific for the temperature of the radiating body, but essentially independent of the material of the body.

The German physicist, Wilhelm Wien was the first to propose an analytical solution for the spectrum of blackbody radiation in 1896. He based his radiation law on the distribution (Maxwell-Boltzmann distribution) of the thermal energy of molecules in the radiating body – by relating the frequency of emitted radiation to the thermal energy of the emitting molecule. As a consequence of such a linkage the intensity of radiation at a specific frequency became proportional to the density of molecules with the corresponding thermal energy. Wilhelm Wien's radiation law gave an excellent match to the observed blackbody spectrum at the high frequency end of the spectrum but deviated remarkably from the observed spectrum at low frequencies (long wavelengths). Shortly after the introduction of Wien's radiation law, Lord Rayleigh introduced a different radiation law; he applied the equipartition principle of thermodynamics and the average thermal energy of a molecule to the standing waves of harmonic frequencies in a black body cavity – which made the energy density increase linearly with the increasing frequency. Rayleigh's radiation law worked very well at the low frequency part of the spectrum but led to unlimited growth of the intensity, “an ultraviolet catastrophe”, at high frequencies.

Wilhelm Wien's radiation law was based on the properties of the emitters at the blackbody surface; Lord Rayleigh's approach was based on standing waves, like “particles” in the blackbody cavity.

In 1900, Max Planck combined the two approaches with an experimental equation. He justified the solution with the famous hypothesis of energy packages, quanta, which allowed radiation to be studied as

particle-like energy objects with a specific distribution function. At high frequencies, Max Planck's distribution of the quanta approached the Maxwell-Boltzmann distribution of the emitters – at low frequencies it became identical with Lord Rayleigh's distribution. With early support from Albert Einstein's explanation of the photoelectric effect in 1905, Max Planck's quantum obtained the interpretation of a particle-like localized photon. Theoretical reasoning of the thermal distribution function of photons in a blackbody cavity had to wait for 25 years, until the Indian physicist, Satyendra Nath Bose – complemented by Einstein, introduced the distribution function known as the Bose-Einstein distribution.

For several years, Max Planck was quite unhappy with his famous equation, $E = hf$, defining the energy of a quantum. As a serious scientist, he felt it important to understand the basis of the equation and its connection to classical mechanics and the Maxwell equations he highly respected. This was perhaps why he did not publish scientific papers on the black body radiation for many years, nor the equation $E = hf$. Only after almost ten years, when the equation had been praised by several physicists, did Max Planck believe that he had made a fundamental finding with the 1900 “quantum equation”.

Does the Planck equation infringe Maxwell's equations?

A basic question arising from the solution for blackbody radiation was: Is a quantum of radiation an intrinsic property of radiation or is it a property of the emission process?

In the final comments of his Nobel lecture in 1911 Wilhelm Wien stated: ...the Planck theory has not yet been brought into a definite form. In science, the redeeming idea often comes from an entirely different direction, investigations in an entirely different field often throw unexpected light on the dark aspects of unresolved problems. ...

Using antenna theory a quantum of radiation can be seen as the minimum amount of radiation, or the minimum amount of energy ejected into a cycle of radiation by an antenna. An antenna is powered by oscillating electrons – obviously the minimum energy is obtained by a single oscillation cycle of a single electron in a dipole. A general solution of Maxwell's equations for a dipole, radiating at a given frequency, is expressed as the average radiation power. The average power is solved in terms of the dipole moment and the number of electrons oscillating in the dipole.

By relating the dipole length to the wavelength emitted and replacing the electric constant ϵ_0 , generally used in the solution, with the magnetic constant $\mu_0 = 1/\epsilon_0 c^2$, the energy emitted into one cycle of radiation is

$$E_\lambda = \frac{P}{f} = \frac{N^2 e^2 \tilde{z}_0^2 \mu_0 16\pi^4 f^4}{32\pi^2 r^2 c f} \frac{2}{3} 4\pi r^2 = N^2 \left(\frac{\tilde{z}_0}{\lambda} \right)^2 \frac{2}{3} (2\pi^3 e^2 \mu_0 c) f \quad (1)$$

where N is the number of electrons oscillating in the dipole, \tilde{z}_0 is the length of the dipole, λ is the wavelength emitted, f is the frequency of the radiation emitted, e is the unit charge, and c is the velocity of light. The factor $2/3$ is a geometrical factor related to a Hertzian dipole. For a single electron oscillating in the dipole ($N=1$) equation (1) obtains the form

$$E_{\lambda(0)} = A (2\pi^3 e^2 \mu_0 c) f; \quad E_{\lambda(0), A=1.1049} = 1.1049 \cdot 2\pi^3 e^2 \mu_0 c \cdot f = hf \quad (2)$$

where $A = (2/3)(\tilde{z}_0/\lambda)^2$ the geometrical constant characteristic to the dipole. The factor $(2\pi^3 e^2 \mu_0 c)$ has the dimension of the Planck constant, and a numerical value of $5.997 \cdot 10^{-34}$ [kgm/s²]. Assuming a geometrical constant $A = 1.1049$ we get the Planck equation with the numerical value $6.626 \cdot 10^{-34}$ [kgm/s²] for the Planck constant h .

Equation (2) means that a one-wavelength dipole characterized by geometrical constant $A = 1.1049$ emits energy $E = hf$ into one cycle of radiation, which is exactly the form of the Planck equation. Further, the Planck constant is now expressed in terms of the basic electrical constants, the unit charge e , and the magnetic constant μ_0 , and the velocity of light as

$$h = 1.1049 \cdot 2\pi^3 e^2 \mu_0 \cdot c \quad (3)$$

By removing the velocity of light from the Planck constant, we end with the “intrinsic Planck constant” $h_0 = h/c$, which converts Planck’s equation into the form

$$E = h_0 \cdot c \cdot f = \frac{h_0}{\lambda} \cdot c^2 = m_{\lambda(0)} \cdot c^2 \quad (4)$$

where the quantity h_0/λ has the dimension of mass [kg]. The quantity $m_{\lambda(0)} = h_0/\lambda$, defined by equation (4), describes the mass equivalence of a cycle of radiation with wavelength λ , produced by a single oscillation of an electron in the emitting dipole.

The energy of a cycle of radiation in equation (4) is formally equivalent to the rest energy mass m_λ , $E = h_0/\lambda \cdot c^2 = m_\lambda c^2$, where the wavelength corresponding to mass m_λ is the Compton wavelength, $\lambda_{Compton} = h_0/m_\lambda$.

The conclusion is, that the Planck equation is a direct consequence of Maxwell’s equations. A quantum of radiation is a cycle of radiation emitted by a single electron transition in an emitter.

How can an atom serve as one-wavelength dipole for any wavelength emitted?

As assumed by both Wilhelm Wien and Max Planck, blackbody radiation is emitted by the atoms or molecules at the blackbody surface – not least due to the linkage of the energy of a quantum of radiation to the average thermal energy of a molecule, $hf \sim kT$. Atomic dimensions are orders of magnitudes smaller than the wavelengths emitted; how can an atom serve as a one-wavelength dipole for any wavelength emitted? We can hardly find an answer available at the time Max Planck presented his equation.

The concept of space-time, introduced in the theory of relativity shortly after Max Planck’s findings, brought a new perspective; in a time differential dt , any object in space travels distance $ds = c \cdot dt$ in a fourth dimension perpendicular to the three space directions. Accordingly, during one cycle of emission, a point-like atom is extended to distance $ds = c \cdot dT = c/f = \lambda$ in the fourth dimension, which allows the study of an atom as one-wavelength dipole in the fourth dimension – for any wavelength emitted!

Following antenna theory by applying the effective area of an antenna, and the density of active antennas as given by the Maxwell-Boltzmann statistics of the emitters at a blackbody surface, the intensity of radiation emitted follows the Rayleigh-Jeans radiation law at low frequencies as determined by the antenna density, and Wien’s radiation law at high frequencies as limited by the activation of the antennas. Combining the two limitation mechanisms we obtain exactly the Planck radiation law.

Maxwell’s equations are symmetric to emission and absorption. Accordingly, quantizing occurs equally at the emission and the absorption;

Absorption or receiving of a quantum of radiation requires that the frequency (or the wavelength) of the incoming radiation corresponds to the characteristic (resonance) frequency of the antenna, and the energy content of the radiation within the capturing area of the antenna is enough to activate one electron transition in the antenna.

An antenna may be isotropic or directional; a quantum of radiation may be emitted as a wave front spreading to all directions, or as a focused radiation pulse to one space direction.

The fine structure constant and a unified expression of energy

The linkage of the Planck constant to the basic electrical constants reveals the nature of the fine structure constant a originally found by Arnold Sommerfeld, as a purely numerical factor, independent of any physical constant

$$a \equiv \frac{e^2 \mu_0 c}{2h} = \frac{e^2 \mu_0 c}{2 \cdot 1.1049 \cdot 2\pi^3 e^2 \mu_0 c} = \frac{1}{1.1049 \cdot 4\pi^3} \approx \frac{1}{137.0360} \quad (5)$$

The intrinsic Planck constant h_0 and the related mass equivalence of electromagnetic radiation allow a unified expression of the energy of a mass particle and of radiation

The rest energy of mass m :
$$E_m = \frac{h_0}{\lambda_m} c^2 = mc^2 = \hbar_0 k_m c^2 \quad (6)$$

The energy of a cycle of electromagnetic radiation:
$$E_\lambda = N^2 \frac{h_0}{\lambda} c^2 = m_\lambda c^2 = \hbar_0 k_\lambda c^2 \quad (7)$$

The list can be completed with the Coulomb energy of opposite electrical charges:

$$E_C = -N_1 N_2 \cdot a \frac{h_0}{2\pi r} c^2 = -m_e c^2 = \hbar_0 k_r c^2 \quad (8)$$

In equations (6) to (8) $h_0 = h_0/2\pi$ is the reduced “intrinsic Planck constant”, and $k = 2\pi/\lambda$ the wavenumber corresponding to the wavelength λ . The close connection between mass and wavelength illustrates the wave nature of mass, or mass as a wavelike substance for the expression of energy.

Summary

The solution of Planck’s equation as the energy of a cycle of radiation emitted by a single electron transition means that a quantum of radiation, a photon, may be emitted as a wave front spreading uniformly in space or as a focused, localized pulse propagating in a specific direction. The wave presentation of the rest energy of mass is formally equivalent to a photon with the Compton wavelength propagating in the fourth dimension, or a resonator with a standing wave of the Compton wavelength. Motion of such a resonator in a space direction results in, due to Doppler effects on the front and back waves, a summation wave carrying the de Broglie momentum in parallel to the motion of the resonator ¹.

Table I summarizes some of the consequences of the reformulation of the Planck constant and the related unified expression of energy following from the solution based on Maxwell’s equations.

	Quantum as a property of radiation	Quantum as a property of the emission/absorption process
The meaning of the Planck constant	h [kg m ² /s] is a quantum of action - postulated	h_0 [kg m] defines the mass equivalence of radiation - derived from Maxwell’s equations
The de Broglie wave	Once c is a factor in h , also the de Broglie wave is fixed to the velocity of light	Carries the momentum at the velocity of a moving particle (the intuition of de Broglie)
The essence of mass	A property of a particle, and a manifestation of energy	The substance for the expression of energy
The energy of radiation/particles	The energy of radiation $E=hf$ The rest energy of particles $E=mc^2$	Unified: $E = c \mathbf{p}^a = \frac{h_0}{\lambda} c^2 = mc^2$
The effect of the expansion of space	Radiation loses energy when propagating in space (one of the factors creating the need for dark energy)	Radiation conserves its energy but the energy density is diluted.

Table I. Comparison of some properties characteristic to a quantum as a property of radiation and a quantum as a property of the emission/absorption process.

¹ T. Suntola, *The Dynamic Universe, Toward a unified picture of physical reality*, Physics Foundations Society, ISBN 9781461027034, e-book ISBN 978-952-67236-5-5 (2012)