Re-Evaluation of the Scout D Experiment as a Test of Relativity Theory

Tuomo Suntola, Ph.D.
Vasamatie 25, 02630 Espoo, FINLAND

The 1976 Scout D rocket test with hydrogen masers revealed a gravitational blueshift attributable to general relativity and a secondary Doppler effect said to be attributable to special relativity, because it correlated with the relative velocity between rocket and reference Earth station, and not the velocities of spacecraft and Earth station separately. The analysis in this paper examines the prediction of the general relativity theory in more detail, and shows that the reported correlation of the secondary Doppler effect with the relative velocity between spacecraft and Earth station was due to the Doppler cancellation system employed — and not, as reported, a manifestation of special relativity. The re-interpretation of the Scout D experiment has important implications for the interpretation of observations of electromagnetic radiation made between moving objects, and ultimately calls into question some of the basic assumptions of the special relativity theory.

Introduction

The 1976 Scout D experiment [1] is one of the most ambitious and most frequently cited tests of general relativity theory (GRT) and special relativity theory (SRT). A hydrogen maser clock in a spacecraft was sent by rocket to an altitude of 10 000 km in a nearly vertical trajectory. The frequency of the hydrogen maser traveling at 10 000 km was compared with the reference frequency of a similar maser at rest at the ground station. The comparison was made by monitoring the difference between the frequency of the ground maser and the frequency of the radio signal transmitted from the maser in the spacecraft. The frequency shift was interpreted in terms of the GRT gravitational blueshift due to the altitude of the spacecraft maser and the SRT time dilation shift, or transverse Doppler effect, or secondary Doppler effect, due to the velocity of the spacecraft relative to the Earth station.

Because of the fast motion of the spacecraft, the transmitted frequency was subject to considerable longitudinal Doppler shift in addition to the effects of interest. The longitudinal Doppler shift was eliminated from the signal by subtracting half of the Doppler shift in the two-way reference signal from the ground station maser to the spacecraft and back (see Fig. 1).

The original analysis of the Scout D experiment [2,3] was formulated in the Earth-centered coordinate frame and included the effects of the gravitational blueshift due to the altitude of the spacecraft and the time dilation shift due to the velocities of the spacecraft and the Earth station.

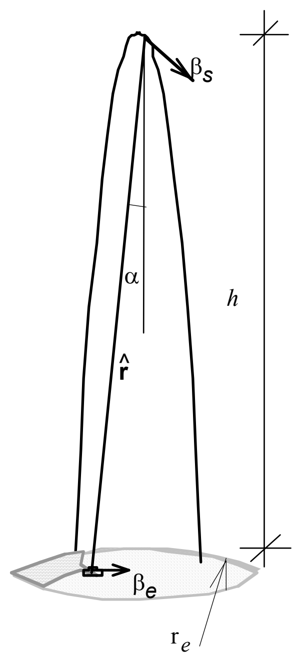

Figure 1a.

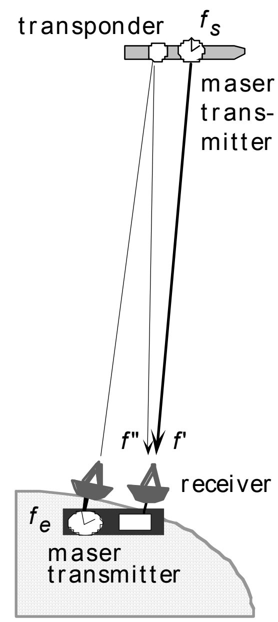

Figure 1b.

Figure 1. **a)** The Scout D experiment for the measurement of the shift of the frequency of a hydrogen maser in a spacecraft. The trajectory of the spacecraft was almost vertical, with maximum altitude h = 10 000 km. A continuous radio signal carrying the maser frequency was transmitted to the Earth station. **b)** In order to eliminate the first order Doppler effect, a signal carrying the frequency of the reference maser at the Earth station was sent to the spacecraft and reflected back by a transponder in parallel with the signal carrying the frequency of the maser in the spacecraft.

More Detailed Analysis

According to the general theory of relativity, the prediction for the combined gravitational shift of the spacecraft maser frequency and the shift due to time dilation received through the radio signal at the Earth station is [3,4]

$$f' = f_e \left[\frac{1+(2\phi_s/c^2)-\beta_s^2}{1+(2\phi_e/c^2)-\beta_e^2} \right]^{1/2} \left(\frac{1-\vec{\beta}_e\cdot\hat{\mathbf{r}}}{1-\vec{\beta}_s\cdot\hat{\mathbf{r}}} \right) \quad (1)$$

where f_e is the proper frequency of the maser at the Earth station, $\hat{\mathbf{r}}$ is the unit vector in the direction of the signal path from the spacecraft to the Earth station, and ϕ_s and ϕ_e are the gravitational potentials of the spacecraft and the Earth station, respectively. The $\vec{\beta}_s$ is the velocity of the spacecraft at the time the signal is sent down to the Earth station, and $\vec{\beta}_e$ is the velocity of the Earth station at the time the signal from the spacecraft is received.

The square root term in Eq. (1) describes the ratio of the proper frequencies of the maser in the spacecraft, f_s, and at the Earth station, f_e:

$$\frac{f_s}{f_e} = \frac{[1+(2\phi_s/c^2)-\beta_s^2]^{1/2}}{[1+(2\phi_e/c^2)-\beta_e^2]^{1/2}} \tag{2}$$

Applying Eq. (2), Eq. (1) can be expressed as

$$\frac{f'}{f_s} = \frac{1-\vec{\beta}_e\cdot\hat{\mathbf{r}}}{1-\vec{\beta}_s\cdot\hat{\mathbf{r}}} \tag{3}$$

The ratio in (3) gives the effect of the velocities of the spacecraft and the Earth station.

For the transponder signal sent from Earth station to the spacecraft and back to the Earth station the corresponding equation is

$$f'' = f_e\left[\frac{1+(2\phi'_e/c^2)-\beta'^2_e}{1+(2\phi_e/c^2)-\beta_e^2}\right]^{1/2}\left(\frac{1-\vec{\beta}_e\cdot\hat{\mathbf{r}}}{1-\vec{\beta}_s\cdot\hat{\mathbf{r}}}\right)\left(\frac{1-\vec{\beta}_s\cdot\hat{\mathbf{r}}'}{1-\vec{\beta}'_e\cdot\hat{\mathbf{r}}'}\right) \tag{4}$$

where β'_e is the velocity of the Earth station at the time the transponder signal was sent from the Earth station to the spacecraft, and $\hat{\mathbf{r}}'$ is the unit vector in the direction of the signal path of the reference signal from the Earth to the spacecraft. The direction of the Earth's velocity vector changes slightly between the transmission and receiving of the transponder signal, so that the direction of β'_e is slightly different from the direction of β_e. The absolute values of β'_e and β_e are the same, however, which makes β'^2_e equal to β_e^2. Because the gravitational potential of the Earth station is also unchanged, $\phi'_e = \phi_e$, Eq. (4) can be expressed as

$$f'' = f_e\left(\frac{1-\vec{\beta}_e\cdot\hat{\mathbf{r}}}{1-\vec{\beta}_s\cdot\hat{\mathbf{r}}}\right)\left(\frac{1-\vec{\beta}_s\cdot\hat{\mathbf{r}}'}{1-\vec{\beta}'_e\cdot\hat{\mathbf{r}}'}\right) \tag{5}$$

which shows that the frequency of the transponder signal is only affected by the Doppler effect.

The frequency measured at the Earth station is

$$\Delta f = (f'_s - f_e) - \tfrac{1}{2}(f'' - f_e) \tag{6}$$

or, when related to the proper frequency of the Earth maser f_e,

$$\frac{\Delta f}{f_e} = \left(\frac{f'_s}{f_e}-1\right) - \frac{1}{2}\left(\frac{f''}{f_e}-1\right) \tag{7}$$

The first term of Eq. (7) can be expressed in the form

$$\begin{aligned}\frac{f'_s}{f_e}-1 &= \frac{f'_s}{f_s}\frac{f_s}{f_e}-1 = \frac{f_s}{f_e}\frac{f'_s}{f_s}-1+\frac{f_s}{f_e}-\frac{f_s}{f_e}\\ &= \left(\frac{f_s}{f_e}-1\right)+\frac{f_s}{f_e}\left(\frac{f'_s}{f_s}-1\right) \approx \left(\frac{f_s}{f_e}-1\right)+\left(\frac{f'_s}{f_s}-1\right)\end{aligned} \tag{8}$$

where f_s is the proper frequency of the maser in the spacecraft as given in Eq. (2) and f'_s the Doppler shifted spacecraft maser frequency given in Eq. (3). The Doppler shift is $f'_s/f_s \approx 10^{-5}$ and the ratio $f_s/f_e \approx 1 \pm 10^{-10}$. Accordingly, the error due to approximation ($\approx 10^{-15}$) made in the last form of Eq. (8) is small enough for the necessary accuracy.

Substituting Eq. (8) into Eq. (7) we get

$$\frac{\Delta f}{f_e} = \left(\frac{f_s}{f_e}-1\right)+\left[\left(\frac{f'_s}{f_s}-1\right)-\frac{1}{2}\left(\frac{f''}{f_e}-1\right)\right] = \frac{\Delta f_s}{f_e}+\frac{\Delta f_D}{f_e} \tag{9}$$

which is equal to the equation used in the analysis of the Scout D experiment by Vessot *et al*. in [3].

Applying Eq. (2), the first term in Eq. (9) can be expressed as

$$\frac{\Delta f_s}{f_e} = \left(\frac{f_s}{f_e}-1\right) = \frac{[1+2\phi_s/c^2-\beta_s^2]^{1/2}}{[1+2\phi_e/c^2-\beta_e^2]^{1/2}}-1 \tag{10}$$

which can be developed by binomial expansion into the form

$$\frac{\Delta f_s}{f_e} = \frac{\phi_s-\phi_e}{c^2}-\frac{1}{2}(\beta_s^2-\beta_e^2) \tag{11}$$

which shows the 'relativistic' shift of the proper frequency of the spacecraft maser due to the difference in the gravitational potentials and velocities of the spacecraft and the Earth station.

The second term in Eq. (9),

$$\frac{\Delta f_D}{f_e} = \left(\frac{f'_s}{f_e}-1\right)-\frac{1}{2}\left(\frac{f''}{f_e}-1\right) \tag{12}$$

is the residual of the Doppler effect, combining the Doppler effect of the spacecraft maser signal and half of the Doppler effect of the two-way Doppler cancellation signal. By applying Eqs. (3) and (5), the Doppler residue can be expressed as

$$\frac{\Delta f_D}{f_e} = \left(\frac{1-\vec{\beta}_e\cdot\hat{\mathbf{r}}}{1-\vec{\beta}_s\cdot\hat{\mathbf{r}}}-1\right)-\frac{1}{2}\left\{\left(\frac{1-\vec{\beta}_e\cdot\hat{\mathbf{r}}}{1-\vec{\beta}_s\cdot\hat{\mathbf{r}}}\right)\left(\frac{1-\vec{\beta}_s\cdot\hat{\mathbf{r}}'}{1-\vec{\beta}'_e\cdot\hat{\mathbf{r}}'}\right)-1\right\} \tag{13}$$

To facilitate the detailed calculation that follows, abbreviations will be applied for the dot products, as follows:

$$A = \vec{\beta}_e\cdot\hat{\mathbf{r}},\ B = \vec{\beta}_s\cdot\hat{\mathbf{r}},\ C = \vec{\beta}_s\cdot\hat{\mathbf{r}}',\ D = \vec{\beta}'_e\cdot\hat{\mathbf{r}}' \tag{14}$$

Substituting A, B, C, D into Eq. (13) we obtain

$$\frac{\Delta f_D}{f_e}=\left(\frac{1-A}{1-B}-1\right)-\frac{1}{2}\left(\frac{1-A}{1-B}\cdot\frac{1-C}{1-D}-1\right) \tag{15}$$

which, neglecting the third and higher order terms, can be expressed as

$$\begin{aligned}\frac{\Delta f}{f_e}&=\tfrac{1}{2}\left[2\left(\frac{1-A}{1-B}-1\right)-\left(\frac{1-A}{1-B}\cdot\frac{1-C}{1-D}-1\right)\right]\\ &\approx\tfrac{1}{2}\Big[2(B+B^2-A-AB)\\ &\qquad-(1+B+B^2-A-AB)(1-C+D+D^2-CD)\Big]\\ &\approx\tfrac{1}{2}\Big(2B+2B^2-2A-2AB-B-B^2+A+AB\\ &\qquad+C-D-D^2+CD+BC-BD-AC+AD\Big)\\ &=\tfrac{1}{2}\Big(-A+B+B^2-AB\\ &\qquad+C-D-D^2+CD+BC-BD-AC+AD\Big)\end{aligned} \tag{16}$$

Observing that a dot product of vectors can be expressed as the sum of the dot products of the components of the vectors

$$\mathbf{a}\cdot\mathbf{b}=(\mathbf{a}_x+\mathbf{a}_y)\cdot\mathbf{b}=\mathbf{a}_x\cdot\mathbf{b}+\mathbf{a}_y\cdot\mathbf{b} \tag{17}$$

We now divide the factors A,B,C,D into dot products expressed as parallel and normal components of $\vec{\beta}_e$ and $\vec{\beta}_s$ with respect to the down-link path in the direction of $\hat{\mathbf{r}}$,

$$\begin{aligned}A&=A_\perp+A_\parallel,\ \ B=B_\perp+B_\parallel\\ C&=C_\perp+C_\parallel,\ \ D=D_\perp+D_\parallel\end{aligned} \tag{18}$$

The dot products $A_\perp,B_\perp,C_\perp,D_\perp$ related to normal components are

$$A_\perp=\vec{\beta}_{e\perp}\cdot\hat{\mathbf{r}}=\beta_{e\perp}\cos(\pi/2)=0 \tag{19}$$

$$B_\perp=\vec{\beta}_{s\perp}\cdot\hat{\mathbf{r}}=\beta_{s\perp}\cos(\pi/2)=0 \tag{20}$$

$$\begin{aligned}C_\perp&=\vec{\beta}_{s\perp}\cdot\hat{\mathbf{r}}'=\beta_{s\perp}\cos(3\pi/2+\Delta\alpha)\\ &=-\beta_{s\perp}\sin(3\pi/2)\sin(\Delta\alpha)\approx\beta_{s\perp}\Delta\alpha\end{aligned} \tag{21}$$

$$\begin{aligned}D_\perp&=\vec{\beta}'_{e\perp}\cdot\hat{\mathbf{r}}'=\beta_{e\perp}\cos(3\pi/2+\Delta\alpha+\Delta\phi)\\ &=-\beta_{e\perp}\sin(3\pi/2)\sin(\Delta\alpha+\Delta\phi)\approx\beta_{e\perp}\Delta\alpha+\beta_{e\perp}\Delta\phi\end{aligned} \tag{22}$$

where

$$\beta_{e\perp}=\beta_e\cos(\alpha) \tag{23}$$

and

$$\begin{aligned}\beta_{s\perp}&=\beta_{sH\alpha}\cos(\alpha)-\beta_{sV\alpha}\sin(\alpha)\\ &=\beta_{sH}\cos(\psi)\cos(\alpha)-\beta_{sV}\sin(\alpha)\cos(\theta)\\ &\quad+\beta_{sH}\sin(\alpha)\sin(\psi)\sin(\theta)\end{aligned} \tag{24}$$

where:

α The angle between the down-link signal path and the normal to the tangent of the path of the Earth station at the time the Doppler cancellation signal is received. The plane determined by the two-way Doppler cancellation signal is called the α-plane.

$\Delta\alpha$ The angle between the down-link and up-link signal paths

$\Delta\phi$ The angle rotated by the Earth during the two-way transmission time of the Doppler cancellation signal

ψ The angle between the horizontal velocities of the spacecraft and the Earth station.

θ The angle between the α-plane and the vertical axis.

$\beta_{e\perp}$ Velocity component of the Earth station perpendicular to the down-link signal path on the α-plane.

$\beta_{s\perp}$ Velocity component of the spacecraft perpendicular to the down-link signal path on the α-plane.

β_{sH} The horizontal velocity of the spacecraft, or more precisely, the velocity of the spacecraft on the plane parallel to the 'base-plane', the tangent plane drawn at the location of the Earth station at the moment the Doppler cancellation signal is received.

$\beta_{sH\alpha}$ The horizontal velocity of the spacecraft on the α-plane (in the direction of the velocity of the Earth station at the moment the Doppler cancellation signal is received).

β_{sV} The vertical velocity of the spacecraft, or more precisely, the velocity of the spacecraft perpendicular to the 'base-plane'.

$\beta_{sV\alpha}$ The component of the vertical velocity of the spacecraft on the α-plane.

(See Figs. 2a to 2d.) Factors $A_\parallel,B_\parallel,C_\parallel,D_\parallel$ can be expressed as

$$A_\parallel=\vec{\beta}_{e\parallel}\cdot\hat{\mathbf{r}}=\beta_{e\parallel}\cos(\pi)=-\beta_{e\parallel} \tag{25}$$

$$B_\parallel=\vec{\beta}_{s\parallel}\cdot\hat{\mathbf{r}}=\beta_{s\parallel}\cos(\pi)=-\beta_{s\parallel} \tag{26}$$

$$\begin{aligned}C_\parallel&=\vec{\beta}_{s\parallel}\cdot\hat{\mathbf{r}}'=\beta_{s\parallel}\cos(0+\Delta\alpha)\\ &=\beta_{s\parallel}\cos(\Delta\alpha)\approx\beta_{s\parallel}[1-\tfrac{1}{2}(\Delta\alpha)^2]\end{aligned} \tag{27}$$

$$\begin{aligned}D_\parallel&=\vec{\beta}'_{e\parallel}\cdot\hat{\mathbf{r}}'=\beta_{e\parallel}\cos(\Delta\alpha+\Delta\phi)\\ &\approx\beta_{e\parallel}[1-\tfrac{1}{2}(\Delta\alpha)^2][1-\tfrac{1}{2}(\Delta\phi)^2]-\Delta\alpha\cdot\Delta\phi\end{aligned} \tag{28}$$

The velocity components of the Earth station and the spacecraft parallel to the signal path are

$$\beta_{e\parallel}=\beta_e\sin(\alpha) \tag{29}$$

and

$$\begin{aligned}\beta_{s\parallel}&=\beta_{sH\alpha}\sin(\alpha)+\beta_{sV\alpha}\cos(\alpha)\\ &=\beta_{sH}\sin(\alpha)\cos(\psi)+\beta_{sV}\cos(\alpha)\cos(\theta)\\ &\quad-\beta_{sH}\cos(\alpha)\sin(\psi)\sin(\theta)\end{aligned} \tag{30}$$

Angles $\Delta\alpha$ and $\Delta\phi$ can be expressed in terms of the two-way signal transmission time Δt

$$\Delta t=2r/c \tag{31}$$

where r is the distance between the Earth station and the spacecraft. Thus,

$$\Delta\alpha=\Delta r_{e\perp}/r=(\beta_{e\perp}c\Delta t)/r=2\beta_{e\perp} \tag{32}$$

where $\Delta r_{e\perp} = \beta_{e\perp} c\Delta t$ is the distance moved by the Earth station perpendicular to the signal path during the two-way transmission time of the Doppler cancellation signal, and

$$\Delta\phi = \beta_e c\Delta t/r_e = 2\beta_e r/r_e \tag{33}$$

Factors A to D can now be expressed as

$$A = A_\perp + A_\parallel = -\beta_{e\parallel} \tag{34}$$

$$B = B_\perp + B_\parallel = -\beta_{s\parallel} \tag{35}$$

$$\begin{aligned} C = C_\perp + C_\parallel &\approx \beta_{s\perp} 2\beta_{e\perp} + \beta_{s\parallel}\ [1 - \tfrac{1}{2}(2\beta_{e\perp})^2] \\ &\approx 2\beta_{s\perp}\beta_{e\perp} + \beta_{s\parallel} \end{aligned} \tag{36}$$

$$\begin{aligned} D = D_\perp + D_\parallel &\approx 2\beta_{e\perp}^2 + 2\beta_e\beta_{e\perp} r/r_e \\ &+\beta_{e\parallel}\{(1-2\beta_{e\perp}^2)[1-2(r/r_e\cdot\beta_e)^2] - 4\beta_{e\perp}\beta_e r/r_e\} \\ &\approx 2\beta_{e\perp}^2 + 2\beta_e\beta_{e\perp} r/r_e + \beta_{e\parallel} \end{aligned} \tag{37}$$

where the third- and higher-order terms have been neglected. The second-order terms are:

$$B^2 \approx \beta_{s\parallel}^2,\ BC \approx -\beta_{s\parallel}^2,\ AD \approx -\beta_{e\parallel}^2,\ D^2 \approx \beta_{e\parallel}^2$$

$$AB = \beta_{e\parallel}\beta_{s\parallel},\ CD \approx \beta_{s\parallel}\beta_{e\parallel} \tag{38-45}$$

$$AC \approx -\beta_{e\parallel}\beta_{s\parallel},\ BD \approx -\beta_{s\parallel}\ \beta_{e\parallel}$$

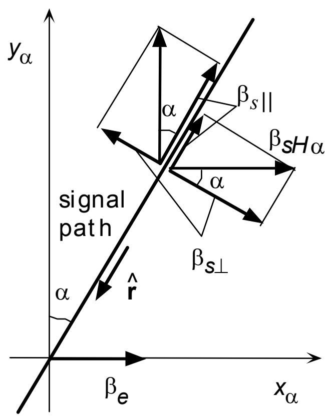

Figure 2a. The components of the velocity of the spacecraft, $\vec{\beta}_s$, on the α-plane, the plane defined by the velocity of the Earth station, $\vec{\beta}_e$, and the down-link signal path from the spacecraft to the Earth station.

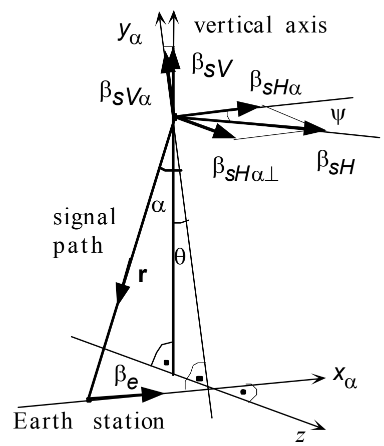

Figure 2b. The α-plane is tilted by angle θ relative to the vertical axis. The coordinate system is that used in the analysis.

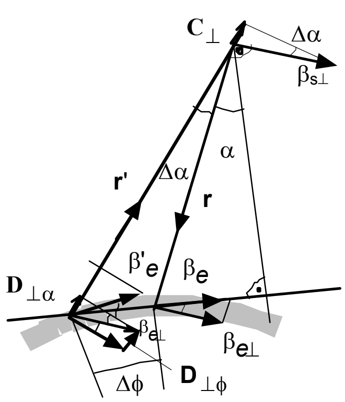

Figure 2c. The two-way Doppler cancellation signal on the α-plane. Velocities $\mathbf{C}_\perp$ and $\mathbf{D}_{\perp\alpha}$ result from angle $\Delta\alpha$ between the up-link and down-link signals. Velocity $\mathbf{D}_{\perp\phi}$ results from velocity $\beta_{\perp e}$ due to the rotation of the Earth by angle $\Delta\phi$.

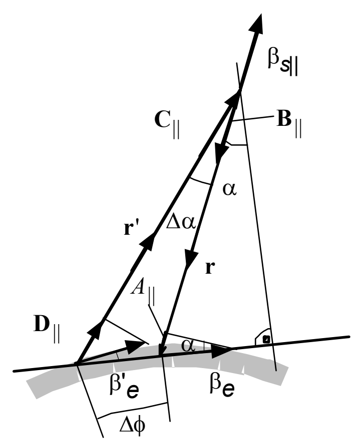

Figure 2d. The two-way Doppler cancellation signal on the α-plane. Velocities $\mathbf{A}_{||}$, $\mathbf{B}_{||}$, $\mathbf{C}_{||}$, and $\mathbf{D}_{||}$ result from velocities $\vec{\beta}_{s\parallel}$ and $\vec{\beta}_{e\parallel}$.

The sum in Eq. (16) is now calculated in groups

$$\Delta f_D/f_e = \tfrac{1}{2}[(B+C-A-D)+(B^2+BC)+(AD-D^2) +(CD-AB-BD-AC)] \quad (46)$$

where

$$B+C-A-D = -\beta_{s\|}+2\beta_{s\perp}\beta_{e\perp}+\beta_{s\|}+\beta_{e\|}-2\beta_{e\perp}^2-\beta_{e\|}-2r/r_e\beta_e\beta_{e\perp} = 2\beta_{s\perp}\beta_{e\perp}-2\beta_{e\perp}^2-2r/r_e\beta_e\beta_{e\perp} \quad (47)$$

$$B^2+BC = \beta_{s\|}^2-\beta_{s\|}^2 = 0 \quad (48)$$

$$AD-D^2 = -\beta_{e\|}^2-\beta_{e\|}^2 = -2\beta_{e\|}^2 \quad (49)$$

$$CD-AB-BD-AC = \beta_{s\|}\beta_{e\|}-\beta_{e\|}\beta_{s\|}+\beta_{e\|}\beta_{s\|}+\beta_{e\|}\beta_{s\|} = 2\beta_{e\|}\beta_{s\|} \quad (50)$$

By substituting Eqs. (47–50) into (46), the term $\Delta f_D / f_e$ related to the Doppler residuals can be expressed as

$$\Delta f_D/f_e = \tfrac{1}{2}(2\beta_{s\perp}\beta_{e\perp}-2\beta_{e\perp}^2-2\beta_e\beta_{e\perp}r/r_e-2\beta_{e\|}^2+2\beta_{e\|}\beta_{s\|}) = \beta_{s\perp}\beta_{e\perp}-\beta_{e\perp}^2-\beta_e\beta_{e\perp}r/r_e-\beta_{e\|}^2+\beta_{e\|}\beta_{s\|} \quad (51)$$

and by applying Eqs. (23), (24), (25), and (30):

$$\begin{aligned}\Delta f_D/f_e &= \beta_e\beta_{s\perp}\cos(\alpha)-\beta_e^2\cos^2(\alpha)-\beta_e^2\cos(\alpha)r/r_e -\beta_e^2\sin^2(\alpha)+\beta_e\beta_{s\|}\sin(\alpha)\\ &= \beta_e\beta_{s\perp}\cos(\alpha)-\beta_e^2[\cos^2(\alpha)+\sin^2(\alpha)] -\beta_e^2\cos(\alpha)r/r_e+\beta_e\beta_{s\|}\sin(\alpha)\\ &= \beta_e[\beta_{s\perp}\cos(\alpha)+\beta_{s\|}\sin(\alpha)]-\beta_e^2-\beta_e^2\cos(\alpha)r/r_e\end{aligned} \quad (52)$$

By applying Eqs. (23) and (30), we can express the terms in square brackets in (52) as

$$\beta_{s\perp}\cos(\alpha) = \beta_{sH\alpha}\cos^2(\alpha)-\beta_{sV\alpha}\cos(\alpha)\sin(\alpha) \quad (53)$$

$$\beta_{s\|}\sin(\alpha) = \beta_{sH\alpha}\sin^2(\alpha)+\beta_{sV\alpha}\sin(\alpha)\cos(\alpha) \quad (54)$$

which give the sum of the terms in the form

$$\beta_{s\perp}\cos(\alpha)+\beta_{s\|}\sin(\alpha) = \beta_{sH\alpha}[\cos^2(\alpha)+\sin^2(\alpha)] = \beta_{sH\alpha} \quad (55)$$

which is the horizontal velocity component of the spacecraft parallel to the velocity of the Earth station, *i.e.* the velocity of the spacecraft in west to east direction (along the x_α -axis in Figure 2b).

By substituting Eq. (55) into Eq. (52) we get

$$\Delta f_D / f_e = \beta_e\beta_{sH\alpha}-\beta_e^2-\beta_e^2\cos(\alpha)r/r_e \quad (56)$$

The last term in Eq. (56) can be expressed in terms of the acceleration **a** of the Earth station due to the rotation of the Earth,

$$\mathbf{a} = \hat{\mathbf{r}}_e v_e^2/r_e = \hat{\mathbf{r}}_e\beta_e^2c^2/r_e = \hat{\mathbf{r}}_e a \quad (57)$$

where $\hat{\mathbf{r}}_e$ is the unit vector in the direction of the radius of the Earth at the Earth station, the local vertical direction. Applying Eq. (57) allows the last term in Eq. (56) to be expressed as

$$r/r_e\ \beta_e^2\cos(\alpha) = \mathbf{r}\cdot\mathbf{a}/c^2 \quad (58)$$

and by applying Eq. (58) allows (56) to be expressed as

$$\Delta f_D/f_e = \beta_e\beta_{sH\alpha}-\beta_e^2-\mathbf{r}\cdot\mathbf{a}/c^2 \quad (59)$$

By substituting Eqs. (11) and (56) for $\Delta f_s / f_e$ and $\Delta f_D / f_e$ in Eq. (9), the frequency difference $\Delta f / f_e$ observed in the experiment can be expressed as

$$\Delta f/f_e = (\phi_s-\phi_e)/c^2-\tfrac{1}{2}(\beta_s^2-\beta_e^2)+\beta_e\beta_{sH\alpha}-\beta_e^2-\mathbf{r}\cdot\mathbf{a}/c^2 \quad (60)$$

Eq. (60) shows the observed frequency difference to be due to four different factors:

1) The difference in the frequencies of the maser in the spacecraft and the maser at the Earth station due to the different gravitational states of the two masers:

$$\Delta f_G/f_e = (\phi_s-\phi_e)/c^2 \quad (61)$$

2) The difference in the shifts of the frequencies of the maser in the spacecraft and the maser at the Earth station due to the different velocities of the masers in the Earth gravitational frame or, in the terms of the theory of relativity, the secondary Doppler effect (the time dilation shift according to general relativity):

$$\Delta f_\beta/f_e = -\tfrac{1}{2}(\beta_s^2-\beta_e^2) \quad (62)$$

3) The effect of angle $\Delta\alpha$ between the up-link and down-link signals on the Doppler cancellation residuals, which appears as the third and fourth terms in Eq. (60):

$$\Delta f_{\Delta\alpha}/f_e = \beta_e\beta_{sH\alpha}-\beta_e^2 \quad (63)$$

where β_e is the speed of the Earth station and $\beta_{sH\alpha}$ is the velocity of the spacecraft in the direction of the velocity of the Earth station.

4) As given in the last term in Eq. (60), the effect of the acceleration of the Earth station, the turn of the velocity of the Earth station, β_e, by angle $\Delta\phi$ due to the rotation of the Earth during the time of two-way transmission of the Doppler cancellation signal:

$$\Delta f_{\Delta\phi}/f_e = -\mathbf{r}\cdot\mathbf{a}/c^2 \quad (64)$$

For further study of the total frequency shift in the Scout D experiment in Eq. (60) we combine the effects $\Delta f_\beta / f_e$ and $\Delta f_{\Delta\alpha} / f_e$ given in Eqs. (62) and (63) as

$$\begin{aligned}\Delta f_\beta / f_e + \Delta f_{\Delta\alpha} / f_e &= -\tfrac{1}{2}(\beta_s^2 - \beta_e^2) + \beta_e \beta_{sH\alpha} - \beta_e^2 \\ &= -\tfrac{1}{2}(\beta_s^2 - \beta_e^2 - 2\beta_e\beta_{sH\alpha} + 2\beta_e^2) \\ &= -\tfrac{1}{2}(\beta_s^2 - 2\beta_e\beta_{sH\alpha} + \beta_e^2)\end{aligned} \tag{65}$$

By breaking $\vec{\beta}_s$ into its orthogonal components $\vec{\beta}_{sH\alpha}$, $\vec{\beta}_{sH\alpha\perp}$, and $\vec{\beta}_{sV}$, we get

$$\Delta f_\beta / f_e + \Delta f_{\Delta\alpha} / f_e = -\tfrac{1}{2}(\beta_{sH\alpha}^2 + \beta_{sH\alpha\perp}^2 + \beta_{sV}^2 - 2\beta_e\beta_{sH\alpha} + \beta_e^2) \tag{66}$$

and by regrouping we obtain

$$\begin{aligned}\Delta f_\beta / f_e + \Delta f_{\Delta\alpha} / f_e &= -\tfrac{1}{2}[(\beta_{sH\alpha}^2 - 2\beta_e\beta_{sH\alpha} + \beta_e^2) + \beta_{sH\alpha\perp}^2 + \beta_{sV}^2] \\ &= -\tfrac{1}{2}[(\beta_{sH\alpha} - \beta_e)^2 + \beta_{sH\alpha\perp}^2 + \beta_{sV}^2]\end{aligned} \tag{67}$$

where the expression in square brackets can be identified as the sum of the squared orthogonal components of the relative velocity between the spacecraft and the Earth station. Accordingly, Eq. (67) can be written in form

$$\Delta f_\beta / f_e + \Delta f_{\Delta\alpha} / f_e = -\tfrac{1}{2}|\vec{\beta}_s - \vec{\beta}_e|^2 \tag{68}$$

which finally authorizes us to express Eq. (60) in the form

$$\Delta f / f_e = (\phi_s - \phi_e)/c^2 - \tfrac{1}{2}|\vec{\beta}_s - \vec{\beta}_e|^2 - \mathbf{r}\cdot\mathbf{a}/c^2 \tag{69}$$

This is exactly the equation used by Vessot *et al*. [1]. Being proportional to the square of the *relative velocity* between the spacecraft and the Earth station, the second term in Eq. (69) *looks like the time dilation term* of special relativity in the Earth station frame of reference. However, the second term in Eq. (69) does not describe the secondary Doppler effect, given in Eq. (62), but rather it results from the combined effects of the secondary Doppler effect and the residuals resulting from the angle Δα between the up-link and down-link signal paths in the Doppler cancellation system employed in the Scout D experiment.

Discussion

The seeming implication of special relativity in the Scout D experiment is not pure coincidence but it reflects the assumptions underlying the special theory of relativity, the linkage of the object to the observer through observation at the velocity of light, which, in the Scout D experiment, in a very concrete way was supplied by the Doppler cancellation signal.

The analysis above confirms that the secondary Doppler effect in the Earth-centered coordinate frame is not related to the relative velocity between the transmitter and the receiver but is due to the difference in the proper frequencies of oscillators moving at different velocities

$$f_A = f_0\sqrt{1-\beta_A^2}\,, \quad f_B = f_0\sqrt{1-\beta_B^2} \tag{70}$$

where f_0 is the frequency of the oscillator at rest in the same gravitational potential. The Scout D experiment, as well as later experience with satellite clocks, confirms that the reference frequency f_0 in Eq. (70) should be regarded as the frequency of a hypothetical clock at rest in the Earth-centered coordinate frame. The state of rest is free from motion due to the rotation of the Earth and any motion relative to the surface of the Earth, which, implicitly, defines the surrounding space the reference at rest.

According to the Doppler terms in Eq. (1), the frequency of electromagnetic radiation transmitted from oscillator *A* moving at velocity $\vec{\beta}_A$ is observed by observer *B* moving at velocity $\vec{\beta}_B$ at the same gravitational potential as

$$f_{A(B)} = f_A(1 - \vec{\beta}_B \cdot \hat{\mathbf{r}})/(1 - \vec{\beta}_A \cdot \hat{\mathbf{r}}) \tag{71}$$

Eq. (71) is formally identical with the classical Doppler equation for a wave transmitted from *A* to *B*, which are objects moving relative to the transmitting medium at velocities $\vec{\beta}_A$ and $\vec{\beta}_B$, respectively. The wavelength counterpart of (71) is

$$\lambda_{A(B)} = \lambda_A(1 - \vec{\beta}_A \cdot \hat{\mathbf{r}})/(1 - \vec{\beta}_B \cdot \hat{\mathbf{r}}) \tag{72}$$

Implicitly, Eqs. (71) and (72) mean that the information about the motion of the source *A* relative to the transmission medium is impeded in the frequency transmitted in direction $\hat{\mathbf{r}}$ (in the frame at rest) as

$$f_A(\hat{\mathbf{r}}) = f_A/(1 - \vec{\beta}_A \cdot \hat{\mathbf{r}}) \tag{73}$$

and in the wavelength

$$\lambda_A(\hat{\mathbf{r}}) = \lambda_A(1 - \vec{\beta}_A \cdot \hat{\mathbf{r}}) \tag{74}$$

propagating at velocity

$$\lambda_A(\hat{\mathbf{r}})\; f_A(\hat{\mathbf{r}}) = \lambda_A f_A = c \tag{75}$$

in the frame at rest.

Combining Eqs. (71) and (73) gives the frequency received by an observer moving at velocity $\vec{\beta}_B$ in the form

$$f_{A(B)}(\hat{\mathbf{r}}) = f_A(\hat{\mathbf{r}})(1 - \vec{\beta}_B \cdot \hat{\mathbf{r}}) \tag{76}$$

which by substituting Eq. (75) for $f_A(\hat{\mathbf{r}})$ takes the form

$$f_{A(B)}(\hat{\mathbf{r}}) = \frac{c}{\lambda_A(\hat{\mathbf{r}})}(1 - \vec{\beta}_B \cdot \hat{\mathbf{r}}) = \frac{c'(\hat{\mathbf{r}})}{\lambda_A(\hat{\mathbf{r}})} \tag{77}$$

which is formally the frequency of radiation with wavelength $\lambda_A(\hat{\mathbf{r}})$ (relevant in the frame at rest) arriving at velocity $c'(\hat{\mathbf{r}})$

$$c'(\hat{\mathbf{r}}) = c(1-\vec{\beta}_B \cdot \hat{\mathbf{r}}) = c - v_B(\hat{\mathbf{r}}) \tag{78}$$

which is the velocity of light minus the velocity of the receiver in the direction the radiation is received.

On the other hand, combining Eqs. (72) and (74) gives the wavelength observed in a frame moving at velocity $\vec{\beta}_B$ in the form

$$\lambda_{A(B)}(\hat{\mathbf{r}}) = \lambda_A(\hat{\mathbf{r}})/(1-\vec{\beta}_B \cdot \hat{\mathbf{r}}) \tag{79}$$

which is observed arriving at velocity

$$c_{A(B)} = \lambda_{A(B)}(\hat{\mathbf{r}}) \cdot f_{A(B)}(\hat{\mathbf{r}}) = c \tag{80}$$

which means that the Doppler shifted frequency $f_{A(B)}(\hat{\mathbf{r}})$ observed in a moving frame is the frequency of radiation with the Doppler shifted wavelength $\lambda_{A(B)}(\hat{\mathbf{r}})$ arriving at velocity c in the moving frame.

When the velocities of the observer and the source are the same, $\vec{\beta}_A = \vec{\beta}_B$, the frequencies and wavelengths received, according to (71) and (72) are

$$f_{A(B)}(\hat{\mathbf{r}}) = f_A\,,\;\; \lambda_{A(B)}(\hat{\mathbf{r}}) = \lambda_A\,,\;\; c_{A(B)} = c \tag{81}$$

which means that the frequency, wavelength, and the propagation velocity observed are independent both of the direction and the absolute value of the motion of the observer and the source relative to the frame at rest, *i.e.* the frequency, wavelength, and the propagation velocity of electromagnetic radiation observed in a moving frame appear exactly as they do in a frame at rest in a gravitational state [5]. Such situation also means zero result in Michelson-Morley type experiments based on the measurement of frequencies or wavelengths of masers or lasers [6,7] instead of a phase shift in a divided beam like in the original experimental setup [8]. The best sensitivity, the ability to detect an ether wind of about 1500 m/s in a phase shift interferometer is reported by Georg Joos in 1930 [9]. The detection limit of 1500 m/s, however, was too high for the detection the rotational motion of the Earth as the motion of the interferometer frame relative to the Earth centered inertial frame.

Acknowledgments

The author thanks Dr. Robert Vessot for supplying the original theoretical analysis of the Scout D experiment.

References

[1] R.F.C. Vessot, M.W. Levine, *et al.*, “Test of Relativistic Gravitation with a Space-Borne Hydrogen Maser”, Phys. Rev. Letters **45** (26) 2081-4 (1980).

[2] D. Kleppner, R.F.C. Vessot, & N.F. Ramsey, “An Orbiting Clock Experiment to Determine the Gravitational Red Shift”, Astrophysics and Space Science **6**, 13-32 (1970).

[3] R.F.C. Vessot, *et.al.*, final report of the 1976 GP-A experiment, private correspondence.

[4] M.D. Harkins, “The Relativistic Doppler Shift in Satellite Tracking”, Radio Science 14 (4) 671-675 (1979).

[5] T. Suntola, **The Dynamic Universe: A New Perspective on Space and Relativity**, ISBN 951-97938-6-0, Suntola Consulting Ltd. (2002), www.sci.fi/~suntola.[6] T.S. Jaseda, *et al.*, “Test of Special Relativity or of the Isotropy of Space by Use of Infrared Masers”, Phys. Rev. **133**, (A5) 1221-1225 (1964).

[6] T.S. Jaseda, *et al.*, “Test of Special Relativity or of the Isotropy of Space by Use of Infrared Masers”, Phys. Rev. **133**, (A5) 1221-1225 (1964).

[7] A. Brillet and J.L. Hall, “Improved Laser Test of the Isotropy of Space” Phys. Rev. Lett. **42** (9) 549-552 (1979).

[8] A.A. Michelson and E.M. Morley, “On the relative motion of the earth and the luminifereous ether”, Am.J.Sci. 3.Ser.34 (203) 333 (1887).

[9] G. Joos, “Die Jenaer Wiederholung des Michelsonversuchs”, Annalen der Physik, **7** (4) 385-407 (1930).