Zero-energy space cancels the need for dark energy

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A detailed analysis of a dynamic solution of the zero-energy condition of gravitation and motion in Einstein's original proposal of spherically closed space shows a precise match to the luminosity–redshift relation of Ia supernovae without dark energy, accelerating expansion, or additional parameters. In such a solution, Einstein's static 4-sphere is allowed to contract or expand instead of being forced to be stationary by means of a cosmological constant, the original formulation of dark energy. In spherically closed space the zero-energy condition determines the mass density and the development of the expansion velocity of space, allowing the derivation of predictions to cosmological observables like the angular size distance, the magnitude, the surface brightness of distant objects, and the orbital velocities in the vicinity of black holes in closed mathematical forms — all with excellent fit with observations without a cosmological constant, dark energy, or accelerating expansion.

In his lectures on gravitation in early 1960's Richard Feynman stated:

"If now we compare this number (total gravitational energy $M_\Sigma^2 G/R$) to the total rest energy of the universe, $M_\Sigma c^2$, lo and behold, we get the amazing result that $GM_\Sigma^2/R = M_\Sigma c^2$, so that the total energy of the universe is zero. — It is exciting to think that it costs nothing to create a new particle, since we can create it at the center of the universe where it will have a negative gravitational energy equal to $M_\Sigma c^2$. — Why this should be so is one of the great mysteries—and therefore one of the important questions of physics. After all, what would be the use of studying physics if the mysteries were not the most important things to investigate".

and further

"One intriguing suggestion is that the universe has a structure analogous to that of a spherical surface. If we move in any direction on such a surface, we never meet a boundary or end, yet the surface is bounded and finite. It might be that our three-dimensional space is such a thing, a tridimensional surface of a four sphere. The arrangement and distribution of galaxies in the world that we see would then be something analogous to a distribution of spots on a spherical ball."

Combining Feynman's "great mystery" of zero-energy space to the "intriguing suggestion of spherically closed space" leads to the concept of Dynamic Universe describing space as a spherically closed structure expanding in the direction of the radius in the fourth dimension — expansion that was unexpected at the time the general relativity was formulated. It was just to prevent the dynamics of spherically closed space that made Einstein to add the cosmological constant to the theory.

A dynamic solution of spherically closed zero-energy space shows the rest energy of matter as the energy of motion mass possesses due to the expansion of space in the fourth dimension and binds the velocity of light in space to the velocity of expansion in the fourth dimension. The time-like fourth dimension of relativity theory becomes replaced by a geometrical fourth dimension showing the direction where zero-energy space propagates the distance $dR_4 = c \cdot dt$ in time differential dt — and thereby creates momentum $\mathbf{p}_4 = m\mathbf{c}_4$ in the fourth dimension for mass at rest in space.

An analysis of the conservation of the total energy in zero-energy space leads to a system of nested energy frames with hypothetical homogeneous space as the universal reference to any local frame in space. Velocity in a local frame becomes related to the local reference at rest. The local kinetic energy becomes related to the rest energy available in the local frame and the gravitational energy due to a local mass center becomes related to the gravitational energy due to the total mass in space. The holistic relativity in zero-energy space is a direct consequence of conserving the zero-energy balance in space. In local frames, the zero-energy approach leads to essentially the same predictions as do the special and general relativity theories, but without relying on modified metrics, Lorentz transformation, the relativity principle, or the equivalence principle. At the cosmological scale, however, the predictions derived from the zero-energy assumption are different — eliminating the need for dark energy or adjustable parameters for consistency with observations.

1. Introduction

Most scientists agree that the physical reality is what it is, and theories serve as our best efforts to describe the reality. The history, however, shows that a prevailing theory may gradually obtain the position of the truth or at least an exclusive way of describing the reality, even in cases the theory has been found defective later. It looks like the more problematic is the prevailing theory, the more defensive appear the attitudes against looking for alternative approaches.

Good examples of undisputable theories are the Maxwell's equations and thermodynamics. In many senses, quantum mechanics earns the same position although the interpretation of the theory, or interpretation of the physical reality behind the theory, have been a subject of active debate since the early days of the introduction of quantum mechanics.

The special theory of relativity – as well as the general theory of relativity – were both under heavy suspicions when the theories were introduced. They both came at a time when there was an urgent need for something in excess to the classical mechanics. The special theory of relativity gave a tolerable explanation to the observations of the velocity of light in moving frames. As a necessary extension, the general theory of relativity generalized the theory to frames in accelerating motion and thereby to the description of gravitation. Gradually, the general theory of relativity gained the position of the prevailing theory of gravitation and together with special relativity an overall replacement of classical dynamics and kinematics.

In spite of the success of the relativity theory in explaining many physical observations, critical discussion on the philosophical basis of the theory still, after 100 years, continues. The variable time and metrics in relativity theory contradicts human conception. Also, the theory is basically observer oriented without a holistic connection to the rest of the universe. The cosmological picture derived from the relativity theory is complex and requires several free parameters for usable predictions. A further principal problem in the standard cosmology model is a violation of the conservation of energy carried by electromagnetic radiation propagating in expanding space. When applied to the microwave background radiation traveled in space since the assumed inflation period, the loss of energy according to the standard model means that about 10% of the total energy in space has been lost due to the redshift of the background radiation (see section 3.9).

Modern observational instruments and space probes have made it possible to see more and more distant space with highly improved accuracy and a variety of details. Last fifteen year's development has challenged the theories perhaps more than the preceding half a century. One of the most striking recent observations is the magnitude versus redshift dependence of supernovae; when interpreted with the standard model, the observations mean that the expansion of space is accelerating instead of decelerating as expected due the work expansion does against gravitation. Observations at high redshifts have also confirmed the Euclidean appearance of distant galaxy space in an obvious disagreement with the prediction of the standard cosmology model.

As a cornerstone of the relativity theory, the relativity principle insists that the laws of nature *shall be written* in such a way that they *look the same* for all observers everywhere, at all times. In relativity theory such a situation is obtained by replacing universal coordinate quantities, time and distance, by locally applied proper time and proper distance. We know from the measurements of radar signals that the velocity of light, which in the relativity theory is a natural constant by definition, slows down close to mass centers in space. When measured in local proper time and distance units, however, it looks the same at any point along the propagation path thus fulfilling the requirement of the relativity principle.

The Dynamic Universe model proposed in this paper is a holistic alternative to the relativity theory. The Dynamic Universe model describes space as a spherically closed structure expanding in the direction of the radius in the fourth dimension — expansion that was unexpected at the time the general relativity was formulated. It was just to prevent the dynamics of spherically closed space that made Einstein to add the cosmological constant to the theory [1].

A holistic approach insists that *the same law of nature* applies at all times, everywhere in space. The Dynamic Universe model does not rely on the relativity principle, Lorentz transformation or equivalence principle. It does not postulate the constancy of the velocity of light. The Dynamic Universe approach postulates the structure of space as a three-dimensional surface of a four-dimensional sphere and relies on a zero-energy balance of the energies of motion and gravitation in space. In such an approach the maximum velocity *in* space becomes fixed to the velocity *of* space in the fourth dimension and the energy of motion mass in space possesses due to the velocity of space in the fourth dimension is observed as the rest energy of matter. Conservation of the total energy in interactions *within space* makes relativity a measure of the locally available share of the total energy [2,3]. The observer as the reference for relativity in the relativity theory becomes replaced by whole space as the reference in the holistic relativity in zero-energy space.

An important cosmological consequence of such a holistic relativity is that, for conserving the zero-energy balance in space, the dimensions of local gravitational systems like galaxies, quasars, as well as the solar system expand in direct proportion to the expansion of space. This is contrary to the prediction in the standard model; the *local conservation of energy* applied in the GR based solutions has led to the conclusion that local gravitational systems conserve their dimensions and the expansion of space occurs as Hubble flow between the local systems only. A consequence of the expansion of local systems together with the overall expansion of the spherically closed space is a Euclidean appearance of the galaxy space (see section 3.4).

As a part of the overall conservation of energy, the zero-energy approach links the energy of a quantum of electromagnetic radiation to the energy carried by a cycle of radiation – a conclusion which can also be derived – without any assumptions tied to the zero-energy approach – from Maxwell's equations (see section 2.5). As expressed by Planck's equation, the higher is the frequency the more energy is pumped into a cycle of radiation at emission. As a requirement of the conservation of the total energy in space, the share of total energy carried by a cycle of radiation is conserved in the course of expansion of space. The increase of the wavelength, the redshift of radiation due to expansion, does not – contrary to the interpretation applied in the derivation of the luminosity distance in the standard cosmology model – result in loss of energy carried by a cycle. The power density of radiation observed in redshifted radiation, however, is decreased because of the increased cycle time accompanying the increased wavelength.

In zero-energy space, the equivalence principle, a cornerstone of general relativity, becomes replaced by the conservation of the total energy, and the field equation based space-time metric of GR space with the geometry of space as an equi-energy surface of the 4-dimensional sphere defining the 3D space. Unlike the Friedman-Lemaître-Robertson-Walker (FLRW) space, zero-energy space has well defined overall geometry which has an essential value on precise, parameter free derivation of predictions to cosmological observables.

Spherically closed space and a zero-energy condition in space are not new ideas. In his lectures on gravitation in early 1960's Richard Feynman considered spherically closed space as an "intriguing suggestion" to the geometry of space and the zero-energy condition between the rest energy and the gravitational energy as "*one of the great mysteries — and therefore one of the important questions of physics*" [4]. The idea of space as spherically closed structure has even longer roots — following the ideas of Georg Bernhard Riemann and Ernst Mach in the 19th century — Albert Einstein in 1917 [1] suggested spherically closed space as the basic cosmological solution for general relativity. Problems, however, arose from the view of static space and the nature of the fourth dimension which already had been defined a time-like dimension.

The Dynamic Universe approach combines the demands of spherically closed structure and the zero-energy condition by the dynamics of the space in the fourth dimension with metric nature. In dynamic space the rest energy of matter obtains the meaning of the energy of motion mass possesses due to the motion of space in the direction of the 4-radius of space and relates all velocities in space to the velocity of space in the fourth dimension.

This paper presents the derivation of the predictions to key cosmology tests in Dynamic Universe with focus in the magnitude versus redshift dependence of standard candles. The predictions obtained are in excellent agreement with observations without hypothetical physical quantities like dark energy. Zero-energy space expands at a decelerating rate until zero at infinity in the future.

2. Physics in zero-energy space

2.1 Gravitation in spherically closed space

If not prevented with a cosmological constant, the zero-energy balance of motion and gravitation in spherically closed space leads to a dynamic solution [3]. Starting from Newtonian type gravitational energy in hypothetical *homogeneous* space, the integration of the gravitational energy of a test mass m throughout mass uniformly distributed in the 3-dimensional surface of a 4-sphere results in gravitational energy

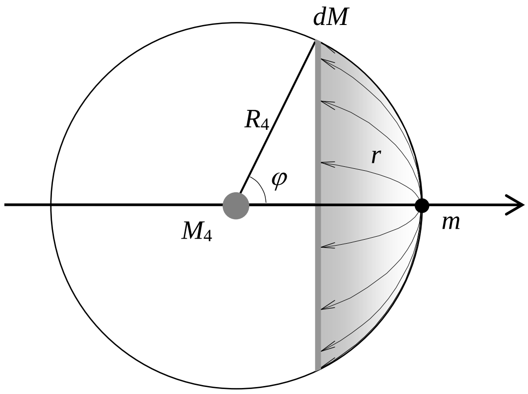


FIG. 2.1-1. The gravitational energy resulting on mass m by mass M_Σ distributed uniformly on the three dimensional surface of a 4-sphere is calculated by integrating the gravitational energy all around the surface. The resulting gravitational energy is equal to the gravitational energy resulting from mass equivalence M_4 at distance R_4 in the direction perpendicular to all three dimensions in spherically closed space. As a consequence of geometrical factors in the 4-sphere, mass equivalence $M_4 = 0.776 \cdot M_\Sigma$.

$$E_g = -\frac{2}{\pi}\frac{GmM_\Sigma}{R_4}\int_0^\pi \frac{\sin^2\phi}{\phi}d\varphi = -\frac{GmM_\Sigma}{R_4}I_4 = -\frac{GmM_4}{R_4} \qquad (2.1{:}1)$$

where $I_4 = 0.776$ is the numerical value of the definite integral in (2.1:1), G is the gravitational constant, M_Σ is the total mass, and R_4 is the radius of the structure in the fourth dimension. Mass $M_4 = I_4 \cdot M_\Sigma$ is the mass equivalence of all mass in spherically closed space. Due to the spherical symmetry, the effect of the mass equivalence is seen at distance R_4 in the fourth dimension, the direction of the local R_4 radius, from any point in space (see Fig.2.1-1).

Accumulation of mass into mass centers in space, by conserving the total gravitational energy, results in local bending of space, which modifies the Newtonian form of gravitational energy.

2.2 The balance of motion and gravitation in hypothetical homogeneous space

Equating the gravitational energy of mass m or the total mass $M_\Sigma = \Sigma m$ to the corresponding rest energy of mass m and M_Σ, we get equations for a zero-energy balance in spherically closed space

$$mc_4^2 - \frac{GmM_4}{R_4} = 0 \qquad \text{or} \qquad M_\Sigma c_4^2 - \frac{GM_\Sigma M_4}{R_4} = 0 \qquad (2.2{:}1)$$

where energy mc^2 is interpreted as the energy equivalence of momentum $m\mathbf{c}$ in the direction of the fourth dimension

$$E_m = c_4|\mathbf{p}_4| = c_4 \cdot m|\mathbf{c}_4| = mc_4^2 \qquad (2.2{:}2)$$

and $\mathbf{c}_4$ the velocity of the structure in the direction of radius R_4. Equation (2.2:2) serves as a fundamental equation defining the energy of motion in space with velocity $\mathbf{c}_4$ in the fourth dimension. As a consequence of the conservation of total energy in interactions in space, the velocity of space in the fourth dimension determines the maximum velocity, and the velocity of light in space. Velocity $c_0 = |\mathbf{c}_4|$ is denoted as the velocity of light in hypothetical homogeneous space. Applying c_0, equation (2.2:2) for the rest energy of matter in space is written as

$$E_{rest} = E_m = c_0|\mathbf{p}_4| = c_0 \cdot m|\mathbf{c}_4| = c_0 mc \qquad (2.2{:}3)$$

where the last form applies also for non-homogeneous space where the momentum in the local fourth dimension may deviate from the $\mathbf{p}_4$ in hypothetical homogeneous space. The local velocity of light in non-homogeneous space is generally denoted as c.

Solved from (2.2:1), velocity c_0 is expressed

$$c_0 = \pm\sqrt{\frac{GM_4}{R_4}} = \pm\sqrt{\frac{0.776 \cdot G\rho 2\pi^2 R_4^3}{R_4}} = \pm 1.246 \cdot \pi R_4 \sqrt{G\rho} \qquad (2.2:4)$$

where the mass density in spherically closed space is $\rho = M_\Sigma/(2\pi^2 R_4^2)$. By applying R_4 = 14 billion light years and by setting the mass density equal to $\rho = 5.0 \cdot 10^{-27}$ [kg/m^3], which is about half of the critical density ρ_0 in the standard cosmology model, velocity c in (2.2:4) obtains the value $c_0 \approx c$ = 300 000 [km/s]. The plus and minus signs in (2.2:4) mean that the zero-energy condition is achieved equally in contraction and expansion of space.

As a fundamental interpretation, equation (2.2:1) shows that the rest energy of mass is built up against release of gravitational energy in a contraction phase before singularity and paid back to gravitational energy in the succeeding expansion phase — in the energy bookkeeping, the rest energy of matter is balanced by an equal energy debt to gravitation (see Fig. 2.2-1).

When solved as a function of time the expansion velocity since singularity becomes

$$c_0 = \frac{dR_4}{dt} = \left(\frac{2}{3} GM''\right)^{1/3} t^{-1/3} \qquad (2.2:5)$$

and the time since singularity becomes

$$t = \int_0^{R_4} \frac{1}{c_0}\, dR_4 = \frac{2}{3}\frac{R_4}{c_0} = \frac{2}{3}\frac{1}{H_0} = 9.3 \cdot 10^9 \qquad [\text{l.y.}] \qquad (2.2:6)$$

The velocity of expansion and, accordingly, the velocity of light decelerate in the course of expansion as

$$\frac{dc_0}{dt} = -\frac{1}{3}\frac{c_0}{t} \qquad (2.2:7)$$

The present deceleration rate of the velocity of expansion is $dc_0/c_0 \approx -3.6 \cdot 10^{-11}$ /year.

A more detailed analysis shows that in zero-energy space the rate of atomic processes, like the characteristic emission and absorption frequencies and radioactive decay occur in direct proportion to the velocity of expansion and, accordingly, to the velocity of light in space. As a result, *the velocity of light is observed as constant* at any time during the expansion. In cosmological observables the faster rate of natural processes is seen, e.g., as a faster rate of radioactive decay – correcting the age estimates of the universe given by radiometric dating – and faster rate of development of galaxy structures in the early universe.

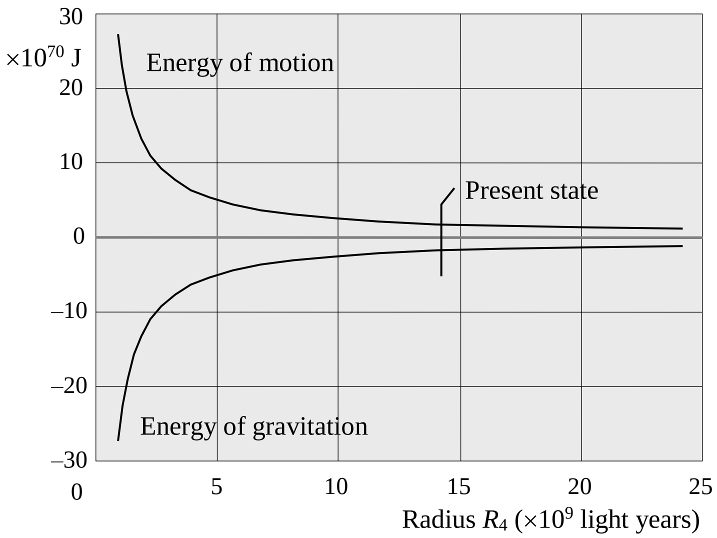


FIG. 2.2-1. The development of the energies of motion and gravitation as functions of the 4-radius in expanding space.

2.3 Universal frame of reference

Spherically closed space allows the definition of a *universal rest frame* as *homogeneous space*, the 3-surface of a 4-sphere with all mass at rest in space and uniformly distributed in the volume. The *state of rest in homogeneous space* means that the only velocity of an object is the velocity given by the expanding R_4 radius in the fourth dimension. Due to the expansion of the radius, objects at rest in different locations in homogeneous space have recession velocities relative to each other. Such velocities are purely kinematic in their nature — there is no momentum in a space direction related to the recession velocity (Fig. 2.3-1).

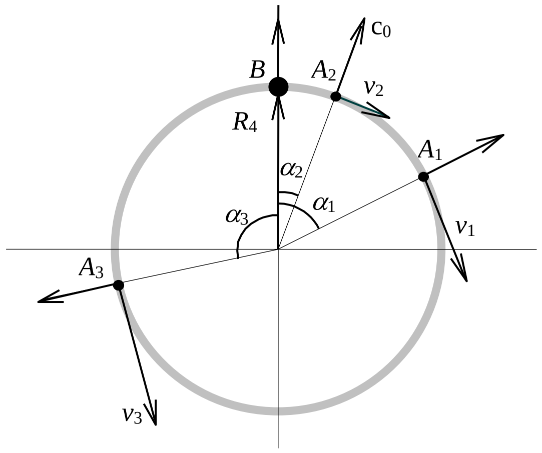


FIG. 2.3-1. The expansion of the 4-radius R_4 causes an increase of all distances in space. The recession velocities v_1, v_2, and v_2 relative to point B are proportional to the distances BA_1, BA_2, and BA_3, respectively. In terms of the distance angle α, distances BA_1, BA_2, and BA_3 can be expressed as a_1R_4, a_2R_4, and a_3R_4.

2.4 Conservables in zero-energy space: mass and energy

Total energy in space

As shown by equation (2.2:1), mass m and M_Σ appear as first order factors both in the expressions of the energy of motion and the energy of gravitation. Without motion or the presence of other mass, no energy or momentum exists. Non-energized mass is not observable. Mass in zero-energy space possesses a highly abstract meaning: rather than a form of energy, it is the *substance for the expression of energy*. Mass is considered as the primary conserved quantity in zero-energy space. The total energy in space is a function of the R_4-radius or the velocity of the expansion of space in the direction of the 4-radius

$$E_{tot} = \left| M_\Sigma \, c_0^2 \right| = \left| -\frac{GM_\Sigma M_4}{R_4} \right| \tag{2.4:1}$$

Mass is assumed to be conserved throughout the development of space from infinity in the past to infinity in the future — the total energy of space is conserved in interactions in space at any momentary value of the R_4-radius and the velocity of expansion.

Energy bookkeeping in zero-energy space — the nested energy frames

In zero-energy space, the state of the expansion, the values of R_4, and c_0 determine the total energy available in space. Conservation of the total energy in interactions in space requires that local energies in space – local kinetic energy, local gravitational energy and electromagnetic energy – are created as part of the total energy determined by the state of expansion. Such a situation is fulfilled in a system of nested energy frames in space. The important outcome of a detailed analysis [3] is that the rest energy of mass m moving at velocity β_n in the n:th of the nested frames can be expressed as

$$E_{rest} = c_0 mc = m_0 c_0^2 \prod_{i=1}^{n} \left(1-\delta_i\right) \sqrt{1-\beta_i^2} \tag{2.4:2}$$

where δ_i is the gravitational factor for mass m in the i:th frame, c_0 is the velocity of light in hypothetical homogeneous space, c is the local velocity of light, and $\beta_i = v_i/c$ is the velocity of m in the i:th frame

$$\delta_i = 1 - \frac{GM_i}{r_{0\delta_i} c_0 c_{0\delta_i}} \approx 1 - \frac{GM_i}{r_i c^2} \quad ; \qquad \beta_i = \frac{v_i}{c_{\delta_i}} = \frac{v_i}{c} \tag{2.4:3}$$

The gravitational factor δ_i is related to the local tilting of space relative to surrounding space as

$$\cos\phi_i = 1 - \delta_i \quad ; \qquad c_\delta = c = c_0 \sum_{i=0}^{n} \left(1 - \delta_i\right) \tag{2.4:4}$$

which also determines the local velocity of light $c = c_\delta$. The effect of motions are described as reduction in the locally available rest mass in the moving object

$$m = m_0 \prod_{i=1}^{n} \sqrt{1 - \beta_i^2} \tag{2.4:5}$$

For mass m at rest in hypothetical homogeneous space all gravitational factors and motions δ_i and β_i are zero and equation (2.4:2) reduces to

$$E_{rest} = m_0 c_0^2 \tag{2.4:5}$$

Mass m on the Earth is subject to velocity β_E and gravitational factor δ_E due to the rotation and the mass of the Earth, respectively. Velocity β_S and gravitational factor δ_S are the velocity and gravitational factors of the Earth in the solar gravitational frame, β_{MW} and δ_{MW} the velocity and gravitational factors of the solar system in the Milky Way frame, etc. As shown by equation (2.4:2), the more energy that is expressed in energy frames in space the smaller the rest energy fueling the internal processes like nuclear decay or characteristic oscillations running atomic clocks in local objects.

Relativity in zero-energy space is not expressed by space-time metrics but by the effects of local motion and gravitation on the locally available rest energy. The local state of rest in zero-energy space is fixed to the zero-momentum state in the energy frame in question.

2.5 Electromagnetic energy and the quantum of radiation

The Coulomb energy

To understand the conservation of mass and energy in zero-energy space, it is necessary to express different forms of energy in formats distinguishing mass or mass equivalence [kg] as the conserved part and the velocity or gravitational distance as the developing part related to c_0, respectively.

Applying the vacuum permeability μ_0, the Coulomb energy for N_1+N_2 unit charges can be expressed in form

$$E_C = \frac{q_1 q_2 \mu_0}{4\pi r} c^2 = N_1 N_2 \frac{e^2 \mu_0}{4\pi r} c^2 = m_C\, c^2 \tag{2.5:1}$$

where the quantity m_C has the dimension of mass [kg]. The *mass equivalence of Coulomb energy*, m_C

$$m_C \equiv N_1 N_2 \frac{e^2 \mu_0}{4\pi r} = N_1 N_2 m_{C(0)} \tag{2.5:2}$$

is conserved when the elementary charge e, numbers N_1, N_2, the vacuum permeability μ_0, and distance r between the two charges are conserved. Quantity $m_{C(0)}$ in the last form of (2.5:2) is the mass equivalence of Coulomb energy for two unit charges at distance r from each other. As shown by equation (2.5:1) the Coulomb energy is conserved relative to the total energy in space at any momentary value of c_0.

Electromagnetic radiation

The standard solution of Maxwell's equations for the power [W] of electromagnetic radiation emitted by a dipole can be written in form

$$P = \left\langle \frac{dE}{dt} \right\rangle = \int_s c\,\mathrm{E}_{\text{ave}} dS = \frac{\Pi_0^2 \mu_0 \omega^4}{32\pi^2 r^2 c} \int_s \sin^2\theta\, dS = \frac{\Pi_0^2 \mu_0 \omega^4}{12\pi c} = \frac{N^2 e^2 z_0^2 \mu_0 \left(2\pi f\right)^4}{12\pi c} \qquad (2.5:3)$$

where $\Pi_0 = Nez_0$ is the dipole moment with N electrons oscillating in a dipole of length z_0. By regrouping and applying $\lambda = c/f$, equation (2.5:3) can be solved for the *energy flux in one cycle* of radiation as

$$E_\lambda = \frac{P}{f} = \frac{N^2 e^2 z_0^2 \mu_0 16\pi^4 f^4}{12\pi c f} = N^2 \left(\frac{z_0}{\lambda}\right)^2 \frac{2}{3}\left(2\pi^3 e^2 \mu_0 c\right) f \qquad (2.5:4)$$

Factor 2/3 in (2.5:4) is the ratio between the average power density of the radiation emitted and the power density on the normal plane of the dipole.

Spherically closed zero-energy space is moving at velocity c in the fourth dimension, which means that a point source at rest in space can be regarded as one-wavelength dipole in the fourth dimension. Observing that any two space directions form a normal plane relative to the fourth dimension the factor 2/3 in (2.5:4) for a dipole in the fourth dimension is replaced by 1. Accordingly, the energy emitted by a point source in one cycle, as one-wavelength dipole in the fourth dimension ($z_0=\lambda$), is

$$E_\lambda = N^2\left(2\pi^3 e^2 \mu_0 c\right) f = N^2\left(2\pi^3 e^2 \mu_0\right) fc = N^2 \frac{2\pi^3 e^2 \mu_0}{\lambda} c^2 \qquad (2.5:5)$$

where the factor $(2\pi^2 e^2 \mu_0 c)$ has the dimension of Planck's constant h [kg·m^2/s] and a numerical value equal to $0.90506 \cdot h$. For $N = 1$ and corrected with factor $\chi_\lambda = 1/0.90506 = 1.1049$ equation (2.5:5) can be written in the form of Planck's equation and interpreted as the energy of a quantum of radiation [5]

$$E_{\lambda(0)} = \chi_\lambda \cdot \left(2\pi^3 e^2 \mu_0 c\right) f = hf \qquad (2.5:6)$$

This simple analysis presented for a point source does not disclose the physical origin of the factor$\chi_\lambda = 1.1049$. An important message, however, is that a quantum of radiation can be expressed in terms of the energy carried by one cycle of radiation. Another important message of equation (2.5:6) is that the velocity of light c is included as a hidden parameter in Planck's constant h.

By removing the velocity of light, c, from the factor $\chi_\lambda \cdot 2\pi^2 e^2 \mu_0 c$ in (2.5:6), we can define the *intrinsic Planck constant*, h_0, which is conserved in the expansion of space

$$h_0 \equiv \frac{h}{c} = \frac{1.1049 \cdot 2\pi^3 e^2 \mu_0 c}{c} = 1.1049 \cdot 2\pi^3 e^2 \mu_0 = 2.210 \cdot 10^{-42} \qquad \left[\mathrm{kg}\cdot\mathrm{m}\right] \qquad (2.5:7)$$

Applying the intrinsic Planck constant, equation (2.5:6) for a quantum of radiation can be rewritten as

$$E_{\lambda(0)} = hf = h_0 c f = \frac{h_0}{\lambda} c^2 = m_{\lambda(0)} c^2 \qquad (2.5:8)$$

where the quantity $h_0/\lambda = m_{\lambda(0)}$ has the dimension of mass [kg] and is referred to as the *mass equivalence of a quantum of electromagnetic radiation*.

Equation (2.5:6) can be generalized to the energy of a cycle of electromagnetic radiation from any electric dipole by inserting the intrinsic Planck constant back to equation (2.5:4)

$$E_\lambda = \frac{P}{f} = N^2\left[\left(\frac{z_0}{\lambda}\right)^2 \frac{2}{3}\frac{1}{\chi_\lambda}\right]\frac{h_0}{\lambda}c_0 c = N^2 A\cdot\frac{h_0}{\lambda}c_0 c = c_0 m_\lambda c \tag{2.5:9}$$

where constant A is determined by the length and the radiation geometry of the dipole. The difference between c_0 and c [see equation (2.4:4)] has been added to (2.5:9). Based on the current knowledge of the gravitational environment of the Earth and the solar system, the velocity of light c on the Earth is of the order of one ppm lower than the velocity of light c_0 in hypothetical homogeneous space. At cosmological distances the velocity of light is approximated as $c \approx c_0$.

The wavelength of electromagnetic radiation propagating in expanding space is subject to lengthening in direct proportion to the expansion. Conservation of the energy of a quantum of radiation, or the energy carried by a cycle of radiation in relation to the total energy in space, requires that the mass equivalence of radiation, $m_{\lambda r} = m_{\lambda e} = h_0/\lambda_e$, is conserved in the course of the propagation of radiation in expanding space

$$E_\lambda = m_{\lambda_e} c_0^2 = \frac{h_0}{\lambda_e} c_0^2 = m_{\lambda_r} c_0^2 \tag{2.5:10}$$

where $m_{\lambda e}$ and $m_{\lambda r}$ are the mass equivalences of radiation at the time the radiation is emitted and received, respectively. When receiving the radiation, the power density observed, however, is reduced due to the increase of the wavelength and the cycle time with the expansion of space.

The concept of mass equivalence of the wavelength can be applied in a reversed form as the wavelength equivalence of mass, i.e. mass can be presented in wave-like form. The energy of the entire mass moving at velocity c_0 in the direction of R_4 described as radiation like energy is

$$E_m = c_0^2 M_\Sigma = \frac{h_0}{\lambda_{M_\Sigma}} c_0^2 \tag{2.5:11}$$

When applied to mass objects moving in space the wavelength equivalence of mass in the direction of the motion in space is equal to the de Broglie wavelength, and in the local fourth dimension, equal to the Compton wavelength.

Applying equations (2.5:6) and (2.5:7) for Planck's constant, the fine structure constant α obtains the form

$$\alpha \equiv \frac{e^2}{2h\varepsilon_0 c} = \frac{e^2\mu_0 c}{2h} = \frac{e^2\mu_0 c}{2\cdot 1.1049\cdot 2\pi^3 e^2\mu_0 c} = \frac{1}{2\cdot 1.1049\cdot 2\pi^3} \approx \frac{1}{137.035} \tag{2.5:12}$$

illustrating the very basic nature of the fine structure constant as a purely numerical or geometrical factor independent of any physical constant. It is important to see that the fine structure constant is independent of the velocity light, which is not constant in zero-energy space.

2.6 Hydrogen like atoms

Applying the expression for rest energy, the first form of (2.4:2), the standard non-relativistic solution of energy states of electrons in a hydrogen-like atom is

$$E_{Z,n} = R\cdot hc\left(\frac{Z}{n}\right)^2 = \frac{\alpha^2}{2}\left(\frac{Z}{n}\right)^2 c_0 m_e c \tag{2.6:1}$$

where R is Rydberg's constant, m_e (or $m_e/(1+m_e/M_N)$) is the mass of an electron, e is the unit charge of electron, Z is the number of protons in the atom, and n is a positive integer. Substituting the effects of gravitation and motion, the last form of (2.4:2), in equation (2.6:1) for the rest energy of an electron in the nucleus energy frame, equation (2.6:1) obtains the form

$$E_{Z,n} = \frac{\alpha^2}{2}\left(\frac{Z}{n}\right)^2 m_0 c_0^2 \prod_{i=1}^{n}(1-\delta_i)\sqrt{1-\beta_i^2} \tag{2.6:2}$$

Balmer's equation for characteristic emission and absorption frequencies solved from (2.6:2) becomes

$$f_{(n1,n2)} = \frac{\Delta E_{(n1,n2)}}{h_0 c} = f_{0(n1,n2)} \prod_{i=1}^{n}(1-\delta_i)\sqrt{1-\beta_i^2} \tag{2.6:3}$$

which shows the effect of motion and gravitation on the frequency. For clocks on the Earth, frame $i = n$ is the Earth gravitational frame, $i = n–1$ is the solar gravitational frame, $i = n–2$ is the Milky Way gravitational frame, etc. In the Earth gravitational frame velocity β_n of a stationary clock is the rotational velocity of the Earth, velocity β_{n-1} is the orbital velocity of the Earth in the Solar frame, β_{n-2} in the Milky Way frame, etc.

Substitution of (2.2:5) for c_0 shows the development of frequency as the function of time since singularity

$$f_{0(n1,n2)} = Z^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]\frac{\alpha^2 m_{e(0)}}{2h_0}\left(\frac{2}{3}GM''\right)^{1/3} t^{-1/3} \tag{2.6:4}$$

The characteristic wavelength corresponding to frequency (2.6:3) is

$$\lambda_{(n1,n2)} = \frac{c}{f_{(n1,n2)}} = \frac{2h_0}{Z^2\left[1/n_1^2 - 1/n_2^2\right]\alpha^2 m_{e(0)}\prod_{i=1}^{n}\sqrt{1-\beta_i^2}} = \frac{\lambda_{0(n1,n2)}}{\prod_{i=1}^{n}\sqrt{1-\beta_i^2}} \tag{2.6:5}$$

Applying the standard solution for the Bohr radius and equation (2.4:5) for the rest mass, the radius of the hydrogen atom can be expressed as

$$a_0 = \frac{h_0^2}{\pi\mu_0 e^2 m_{e(N)}} = \frac{h_0}{2\pi\alpha m_{e(0)}\prod_{i=1}^{n}\sqrt{1-\beta_i^2}} = \frac{a_{0(0)}}{\prod_{i=1}^{n}\sqrt{1-\beta_i^2}} \tag{2.6:6}$$

The emission wavelength $\lambda_{(n1,n2)}$ in equation (2.6:5) can be expressed in terms of the Bohr radius $a_{0(0)}$ as

$$\lambda_{(n1,n2)} = \frac{4\pi\, a_{0(0)}}{\alpha Z^2\left[1/n_1^2 - 1/n_2^2\right]\prod_{i=1}^{n}\sqrt{1-\beta_i^2}} = \frac{4\pi a_0}{\alpha Z^2\left[1/n_1^2 - 1/n_2^2\right]} \tag{2.6:7}$$

which shows that the wavelength emitted is directly proportional to the Bohr radius of the atom.

Both the characteristic emission wavelength and the Bohr radius are conserved in the course of the expansion space.

In fact, equation (2.6:7) is just another form of Balmer's formula, which does not require any assumptions tied to zero-energy space. Equation (2.6:7) also means that, like the dimensions of an atom, the characteristic emission and absorption wavelengths of an atom are unchanged in the course of the expansion of space but increase with the velocity of the atom.

3. Cosmological appearance of zero-energy space

3.1 Cosmological principle in zero-energy space

At cosmological scale spherically closed space is isotropic and homogeneous, i.e., it appears the same as seen from any point in space. As a major difference to the FLRW cosmology, local gravitational systems in zero-energy space are subject to expansion in direct proportion to the expansion of the R_4 radius. Accordingly, e.g., the radii of galaxies are not observed as standard rods but as expanding objects which makes the sizes of galaxies appear in Euclidean geometry to the observer. In the Earth gravitational frame, the linkage of orbital radii to the expansion of the R_4-radius means that about 2.8 cm out of the 3.8 cm annual increase in the Earth to Moon distance is due to expansion of space and only 1 cm is due to tidal interactions or other mechanisms. Expansion of space is a direct consequence of the zero-energy demand, no separate Hubble flow between galaxies or galaxy groups is assumed.

As shown by the analysis of the Bohr radius, material objects built of atoms and molecules are not subject to expansion with space. As shown by equations (2.6:5) and (2.6:6), like the Bohr radius, the characteristic emission wavelengths of atomic objects are likewise unchanged in the course of the expansion of space. When propagating in space, the wavelength of electromagnetic radiation is increased in direct proportion to the expansion. Accordingly, when detected after propagation in space, characteristic radiation is observed redshifted relative to the wavelength emitted by the corresponding transition in situ at the time of observation.

3.2 The Hubble law

Observations of distant objects are based on the propagation of electromagnetic radiation. Because the 4-radius of space increases at the same velocity as light propagates in the tangential (space) direction, the actual path of light is a spiral in four dimensions, and the length of the optical path in space (the tangential component of the path) is equal to the increase of the 4-radius of space (the radial component of the path) during the propagation [see Fig. 3.2-1]

$$D = R_{4(2)} - R_{4(1)} \tag{3.2:1}$$

The differential of optical distance can be expressed in terms of R_4 and the distance angle α as

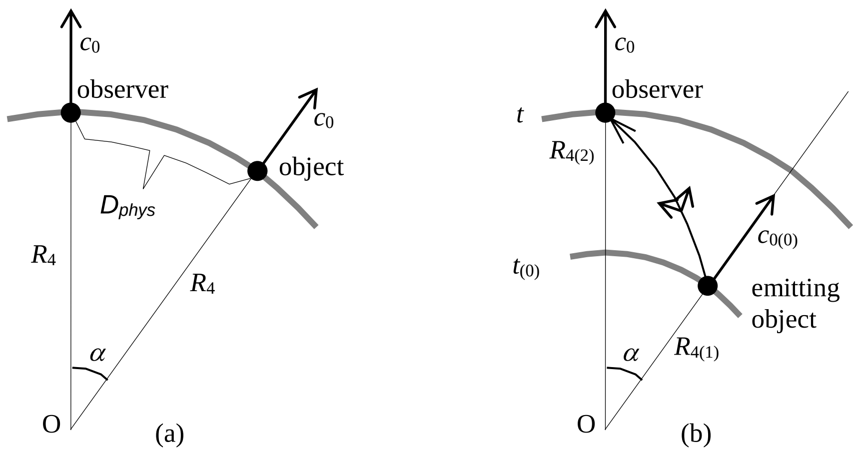


FIG. 3.2-1. (a) The classical Hubble law corresponds to Euclidean space where the distance of the object is equal to the physical distance, the arc D_{phys}, at the time of the observation. (b) When the propagation time of light from the object is taken into account the optical distance is the length of the integrated path light propagates in space, in the tangential direction in the 4-sphere $D_{opt} = D = \int dD_{\perp}$. Because the velocity of light in space is equal to the expansion of space in the direction of R_4, the optical distance is $D = R_{(2)} - R_{(1)}$, the lengthening of the 4-radius during the propagation time.

$$dD = R_4 d\alpha = c_0 dt = dR_4 \tag{3.2:2}$$

By first solving for the distance angle α

$$\alpha = \int_{R_{4(0)}}^{R_4} \frac{dR_4}{R_4} = \ln \frac{R_{4(2)}}{R_{4(1)}} = \ln \frac{R_4}{R_4 - D} \tag{3.2:3}$$

the optical distance D obtains the form

$$D = R_4 \left(1 - e^{-\alpha}\right) \tag{3.2:4}$$

where $R_4 = R_{4(2)}$ means the value of the 4-radius at the time of the observation.

The observed recession velocity, the velocity at which the optical distance increases, obtains the form

$$v_{rec(optical)} = \frac{dD}{dt} = c_0 \left(1 - e^{-\alpha}\right) = \frac{D}{R_4} c_0 \tag{3.2:5}$$

As demonstrated by equation (3.2:5) the maximum value of the observed (optical) recession velocity never exceeds the velocity of light, c, at the time of the observation, but approaches it asymptotically when distance D approaches the length of 4-radius R_4.

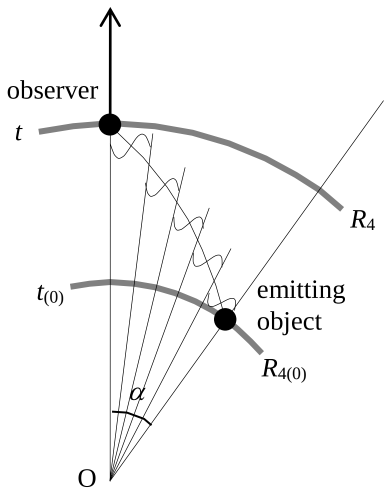


FIG. 3.3-1. The redshift of radiation results from the lengthening of the wavelength with the expansion of space. The number of quanta (or wavelengths of radiation) on the way from an emitter of constant intensity to an observer at a fixed distance angle from the emitter is constant with time.

3.3 Redshift of wavelength and the angular distance

As stated in the previous section the characteristic emission wavelengths of atoms are unchanged in the course of the expansion of space. The wavelength of radiation propagating in expanding space is assumed to be subject to increase in direct proportion to the expansion. Accordingly, redshift becomes

$$z = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{R_4 - R_{4(0)}}{R_{4(0)}} = \frac{D/R_4}{1 - D/R_4} = e^{\alpha} - 1 \tag{3.3:1}$$

where $D = R_4 - R_{4(0)}$ is the optical distance of the object given in (3.2:4), λ and R_4 are the wavelength and the 4-radius at the time of the observation, respectively, and $R_{4(0)}$ is the 4-radius of space at the time the observed light was emitted (see Fig. 3.3-1).

From equation (3.3:1), the optical distance can be expressed in terms of the redshift as

$$D = \frac{z}{1+z} R_4 = R_4 \left(1 - e^{-\alpha}\right) \tag{3.3:2}$$

(see Fig. 3.3-2).

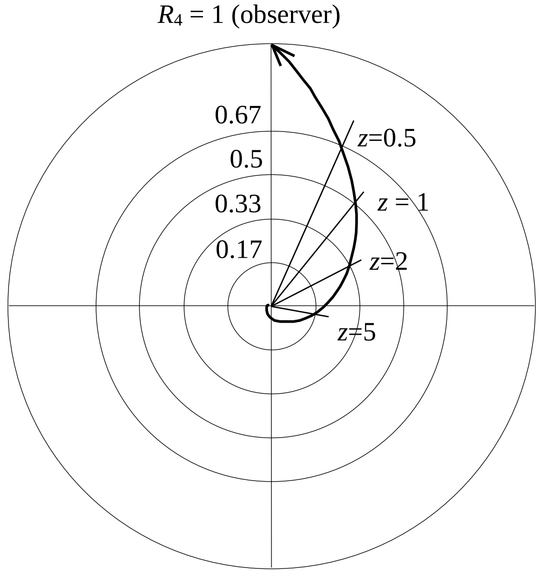


FIG. 3.3-2. Expansion of space during the propagation time of light from objects at different distances: The length of the 4-radius R_4 and the corresponding optical path is indicated for redshifts z = 0.5 to 5.

The optical distance D of equation (3.3:2) corresponds to the angular size distance in the standard model [6]

$$D_A = \frac{c/H_0}{(1+z)} \int_0^z \frac{1}{\sqrt{(1+z)^2 (1+\Omega_m z) - z(2+z)\Omega_\Lambda}} dz \tag{3.3:3}$$

where the flat space condition ($\Omega_m + \Omega_\Lambda = 1$) is assumed, and $c/H_0 = R_H$ is the Hubble radius corresponding to R_4 in zero-energy space. Ω_m and Ω_Λ give the shares of the densities of baryonic plus dark mass and the dark energy in space, respectively.

Figure 3.3-3 compares the angular size distance (3.3:3) of the standard model and the optical distance of equation (3.3:2) in zero-energy space. The angular size distance of FLRW space turns to decrease at redshifts above $z \approx 2$ but the optical distance of equation (3.3:2) in zero-energy space approaches asymptotically the 4-radius R_4 at high redshifts.

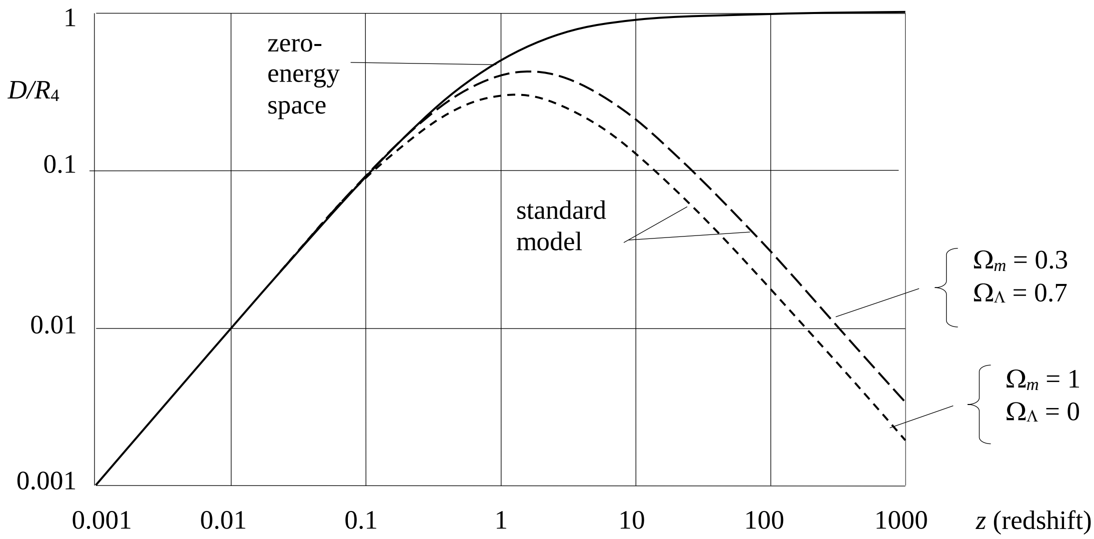


FIG. 3.3-3. The angular size distance (optical distance) of objects in zero-energy space (3.3:2) (solid line), and the angular size distance in FLRW space (3.3:3) for Ω_m = 0.3, Ω_Λ = 0.7 and for Ω_m = 1, Ω_Λ = 0, corresponding to the present estimates of mass and dark energy densities in ΛCDM corrected space, and the Einstein-deSitter condition in FLRW space, respectively (dashed lines).

3.4 Angular size of a standard rod and expanding objects

Applying the optical distance D in equation (3.3:2), the angular size of an *expanding object* with diameter $d = d_R/(1+z)$ at the time light from the object is emitted is

$$\theta = \frac{d}{D} = \frac{d_R}{(1+z)}\frac{(1+z)}{R_4 z} = \frac{d_R}{R_4}\frac{1}{z} = \frac{\alpha_d}{z} \quad ; \quad \frac{\theta}{d_R/R_4} = \frac{\theta}{\alpha_d} = \frac{1}{z} \tag{3.4:1}$$

where the ratio $d_R/R_4 = \alpha_d$ means the angular size of the expanding object as seen from the center of the 4-sphere. Equation (3.4:1) implies a Euclidean appearance of expanding objects in space (see Fig. 3.4-1).

The observation angle of a standard rod or *non-expanding objects* (solid objects like stars) is

$$\theta = \frac{d_{rod}}{D} = \frac{d_{rod}}{R_4}\frac{(1+z)}{z} \quad ; \quad \frac{\theta}{d_{rod}/R_4} = \frac{(1+z)}{z} \tag{3.4:2}$$

As shown by equation (3.4:2), the observation angle of a standard rod approaches the size angle $\alpha_d = d_{rod}/R_4$ at high redshift ($z >> 1$).

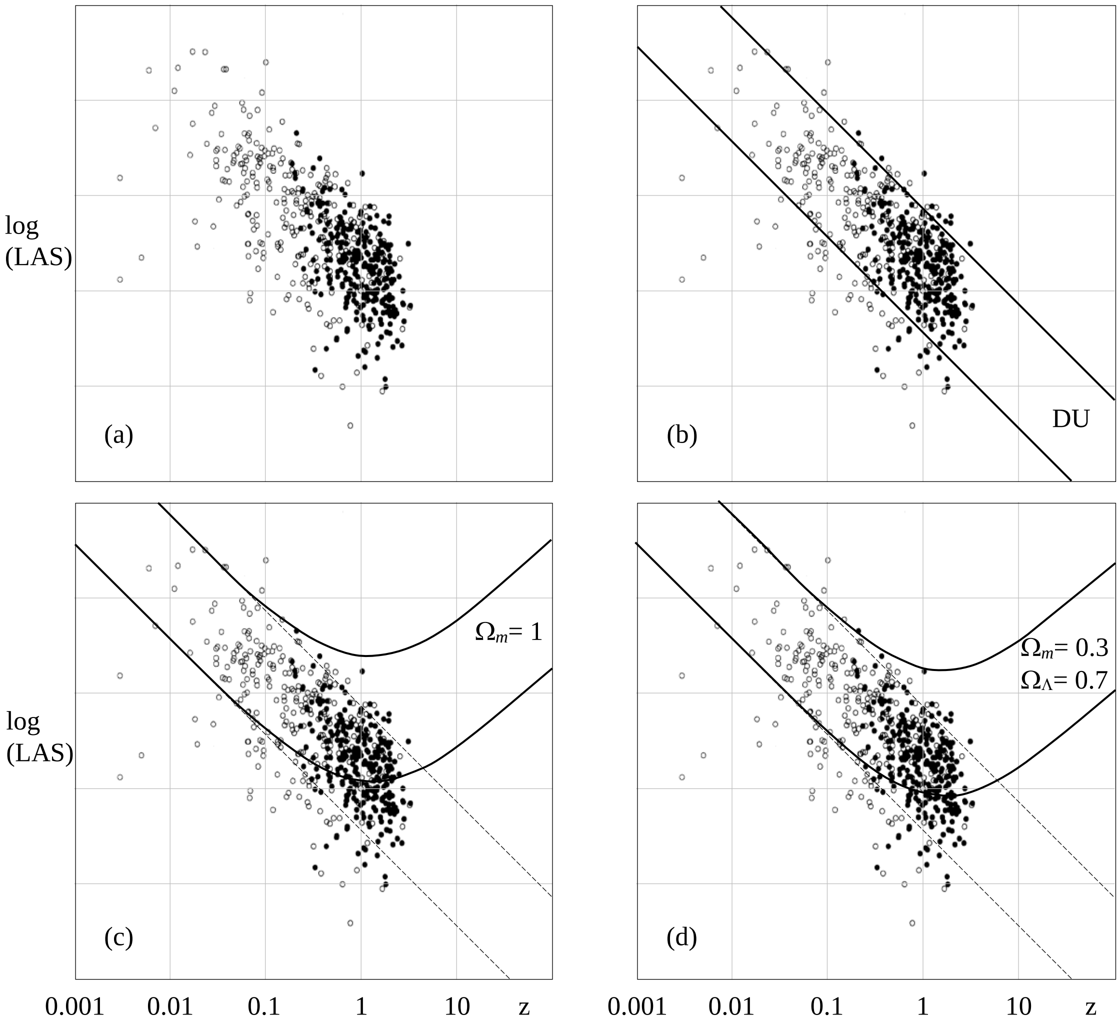


FIGURE 3.4-1. (a) Largest apparent size (LAS) vs. redshift for a sample of 540 Fanaroff-Riley type II double radio sources collected by Nilsson *et al*. [7]. Open circles represent galaxies; filled circles quasars. (b) The angular size of the objects in Dynamic Universe follows closely the Euclidean 1/z prediction given in equation (3.4:1) for expanding objects. Open circles in the figure represent galaxies; filled circles, quasars. (c) The angular size prediction based on the Einstein – de Sitter solution of the standard model. According to the prediction the angular sizes of objects with redshifts above z > 1 show an increasing trend. (d) The angular size prediction of the standard model with $\Omega_\Lambda = 0.7$ and $\Omega_m = 0.3$. The turning point to an increasing angular size is shifted to a slightly higher redshift. No evolution of the objects is assumed in the predictions.

3.5 The effects of redshift and distance on electromagnetic radiation

As shown by equations (2.5:1) and (2.5:9), the Coulomb energy and the energy of electromagnetic radiation can be expressed in terms of a mass equivalence and the velocity of light, formally, like the rest energy of matter.

Matter: $$E_{rest} = mc_0c \qquad (3.5:1)$$

Cycle (N^2 quanta) of electromagnetic radiation: $$E_\lambda = N^2 \frac{h_0}{\lambda_e} c_0c = m_\lambda c_0 c \qquad (3.5:2)$$

Coulomb energy: $$E_C = N_1N_2\alpha \frac{h_0}{2\pi r} c_0c = m_C c_0 c \qquad (3.5:3)$$

The zero-energy condition is conserved when all forms of energy are conserved relative the total energy of space $E_{tot} = M_\Sigma c_0^2$, which is fulfilled when mass or mass equivalence is conserved. The mass equivalence of an electromagnetic wave is determined by the Planck equation *at emission*; when propagating in expanding space the mass equivalence is conserved but diluted in density due to the lengthening of the wave. The observed energy flux, or power density [W/m^2] of redshifted radiation when received at wavelength $\lambda_r = \lambda_e(1+z)$ is

$$F_{rec} = E_\lambda f_r = m_{\lambda e}cc_0 \cdot f_r = \frac{h_0}{\lambda_e}cc_0 \cdot \frac{c}{\lambda_r} = \frac{h_0}{\lambda_e}\frac{c_0c^2}{\lambda_e(1+z)} = \frac{h_0c_0c^2}{\lambda_e^2(1+z)} \qquad (3.5:4)$$

where λ_e is the wavelength of radiation at the emission. The reference flux emitted by an identical source at the time and location the redshifted radiation is received ($\lambda_r = \lambda_e$) is

$$F_{emit(ref)} = E_\lambda f = \frac{h_0}{\lambda_e}cc_0 f = m_{\lambda_e}cc_0\frac{c}{\lambda_e} = \frac{h_0}{\lambda_e}\frac{c_0c^2}{\lambda_e} = \frac{h_0c_0c^2}{\lambda_e^2} \qquad (3.5:5)$$

Relative to the reference flux, the power density in the redshifted flux is

$$F_{rec} = \frac{F_{emit(ref)}}{(1+z)} \qquad (3.5:6)$$

In zero energy space, the energy flux observed in radiation redshifted by z is diluted by factor (1+z), not by factor $(1+z)^2$ as assumed in the standard model solution [8]. The difference comes from the interpretation of the effect of redshift on the energy of a quantum. As first proposed by Hubble and Humason [9] and later by de Sitter [10], the energy of a quantum is reduced by (1+z) as a consequence of the effect of Planck's equation $E = hf$ as a reduction of the "intensity of the radiation". When receiving the redshifted radiation at a lowered frequency, a second (1+z) factor was assumed. Hubble [11,12] considered that the latter is relevant only in the case that the redshift is due to recession velocity. The first (1+z) factor was called the "energy effect" and the second (1+z) factor the "number effect".

Conservation of the mass equivalence of radiation in zero-energy space negates the basis for an "energy effect" as a violation of the conservation of energy. The analysis of the linkage between Planck's equation and Maxwell's equations in Section 2.5 shows that Planck's equation describes the energy conversion at the *emission* of electromagnetic radiation. Redshift should be understood as *dilution of the energy density due to an increase in the wavelength* in the direction of propagation, not as *losing of energy*. Accordingly, the observed energy flux $F = E_\lambda f$ is subject only to a single (1+z) dilution factor, the "number effect" in the historical terms.

Referring to equation (3.5:4), at distance D from source A the density of the energy flux F_A is

$$F_A = \frac{N^2}{4\pi D^2}\frac{h_0c_0c^2}{\lambda_e^2(1+z)} \qquad (3.5:7)$$

where N is the intensity factor of the source. Related to the flux density F_B from a reference source B with same intensity at distance d_0 ($z \approx 0$) the energy flux F_A is

$$F_A = F_B \cdot \frac{N^2}{4\pi D^2} \frac{h_0 c_0 c^2}{\lambda_e^2 (1+z)} \Bigg/ \frac{N^2}{4\pi d_0^2} \frac{h_0 c_0 c^2}{\lambda_e^2} = F_B \cdot \frac{d_0^2}{D^2} \frac{1}{(1+z)} \qquad (3.5:8)$$

Substitution of equation (3.3:2) for D in (3.5:8) gives

$$F_A = F_B \cdot \frac{d_0^2}{R_4^2} \frac{(1+z)^2}{z^2} \frac{1}{(1+z)} = F_B \cdot \frac{d_0^2}{R_4^2} \frac{(1+z)}{z^2} \qquad (3.5:9)$$

For a d_0 = 10 pc reference source, $F_B = F_{10pc}$ we get the expression for the apparent magnitude

$$m = M - 2.5 \log \frac{F_A}{F_{10pc}} = M + 5 \log \frac{R_4}{10[\mathrm{pc}]} + 5 \log z - 2.5 \log (1+z) \qquad (3.5:10)$$

where M is the absolute magnitude of the reference source at distance d_0.

Equation (3.5:10) applies for the *bolometric energy flux* observed for radiation from a source at optical distance (angular size distance) $D = R_4 \cdot z/(1+z)$ from the observer in zero-energy space. Equation (3.5:10) does not include possible effects of galactic extinction, spectral distortion in Earth atmosphere, or effects due to the local motion and gravitational environment of the source and the receiver.

3.6 Multi-bandpass detection

Multi-bandpass photometry allows a precise analysis of the spectral distribution of the radiation source and makes it possible to follow the spectral shift due to redshift. Figure 3.6-1 demonstrates the effect of redshift on the spectral shift and radiation power of a black body source with zero redshift maximum at the nominal wavelength of the V (visual) filter. The bolometric radiation power is a function of the redshift

$$F_{bol} = \int_0^\infty F\left(\frac{d\lambda_z}{\lambda_z}\right) = \frac{F_{bol(z=0)}}{1+z} \qquad (3.6:1)$$

At redshift z = 1 the wavelength of the maximum has doubled and the optimum filter is Y , at z = 1.8 the maximum power is obtained in filter H. By matching a filter to the peak wavelength of the spectrum at each redshift, the detected power density at each filter is in direct proportion to the bolometric energy flux of the radiation (assuming that the relative widths of the filters are equal).

Assuming a blackbody source and filter X with a transmission function equal to normal distribution with half-width W_X, the detected energy flux, normalized to the bolometric flux, can be expressed as

$$F_{X(z)} = \frac{1}{4.780} \frac{F_{bol(z=0)}}{(1+z)} \int_0^\infty \left[\left(\lambda_0 \Big/ \frac{\lambda}{1+z} \right)^4 \Bigg/ \left(e^{\left(\lambda_0 / \frac{\lambda}{1+z}\right)} - 1 \right) \right] e^{-\frac{2.773}{W_X^2} \left(\frac{\lambda/(1+z)}{\lambda_{C(X)}} - 1 \right)^2} \frac{d\lambda}{\lambda} \qquad (3.6:2)$$

Equation (3.6:2) gives the flux observed through filter X as a function of the redshift of the radiation. The constant in front of the integral in equation (3.6:2) matches the energy flow through a filter with nominal width to the bolometric radiation flow. Figure 3.6-2 shows the normalized transmission curves calculated for filters UBVRIZJ by integration of (3.6:2). Each curve touches the bolometric curve (3.6:1) at the redshift matching the maximum of the radiation flux to the nominal wavelength of the filter.

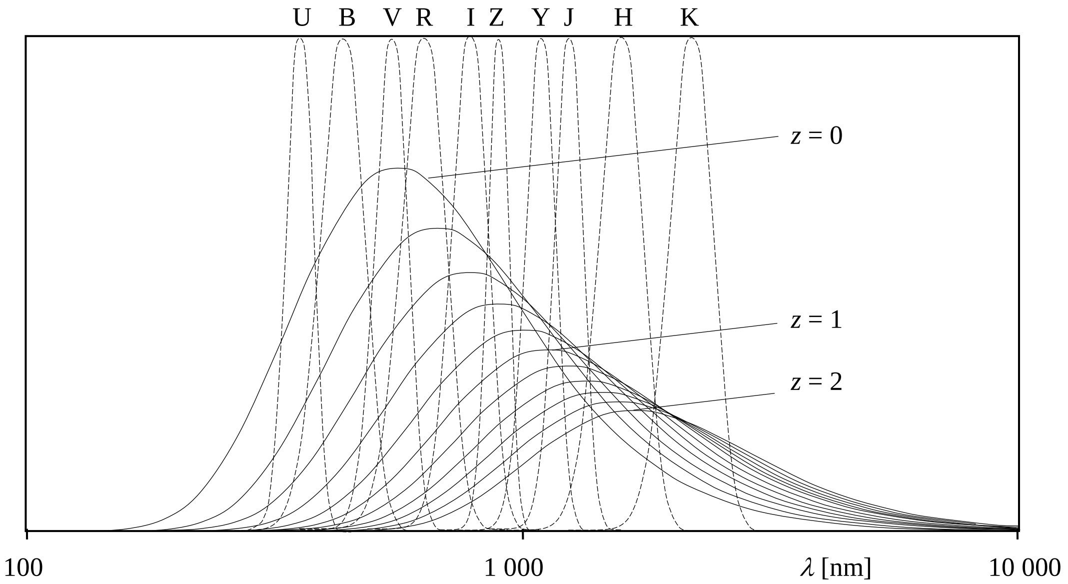


FIG. 3.6-1. The effect of redshift z = 0…2 (shown in steps of 0.2) on the energy flux density per relative bandwidth of the blackbody radiation spectrum from a T = 6600 °K blackbody source corresponding to λ_T = 440 nm and λ_W = 557 nm (solid curves). Transmission curves of UBVRIZYJHK filters are shown with dashed lines. The half widths of the filters follow the widths of standard filters in the Johnson system. All transmission curves are approximated with a normal distribution. The horizontal axis shows the wavelength in nanometers in a logarithmic scale.

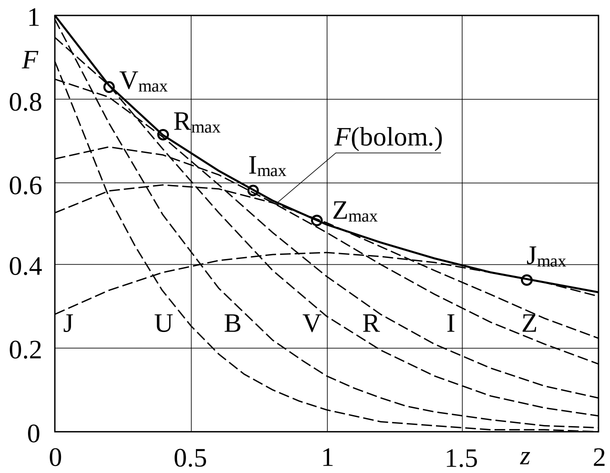
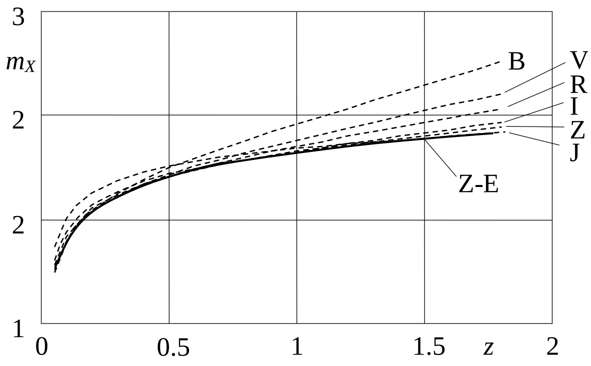


FIG. 3.6-2. Transmission curves obtained by numerical integration of (3.6:2) for filters UBVRIZJ for radiation in the redshift range z = 0…2 from a blackbody with λ_T = 350 nm (λ_W = 440 nm, T = 8300 °K). Each curve touches the bolometric curve of equation (3.6:1) at the redshift matching maximum of the radiation flux to the nominal wavelength λ_W of the filter (small circles in the figure).

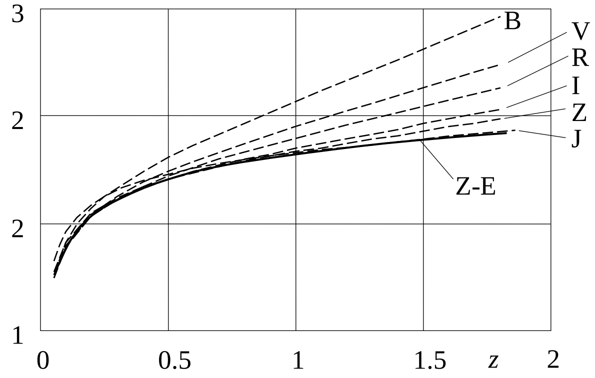
Figures 3.6-3(a,b) illustrate the magnitudes calculated for filters $X = B, V, R, I, Z, J$ from equation (3.5:10) in the redshift range z = 0…2. Each curve touches the red dotted curve corresponding to the bolometric magnitude obtainable with optimal filters in the redshift range studied. Figure 11(c) shows observed magnitudes in BVRIYJ filters as presented by Tonry *et al*. [13] in Table 7.

In the observation praxis, partly for historical reasons, direct observations of magnitudes in the bandpass filters are treated with *K-correction* which corrects the filter mismatch and converts the observed magnitude to the "emitter's rest frame" presented by observations in a bandpass matched to a low redshift reference of the objects studied. The *K-correction* for observations in the X_j band relative to the rest frame reference in the X_i band is defined [14]

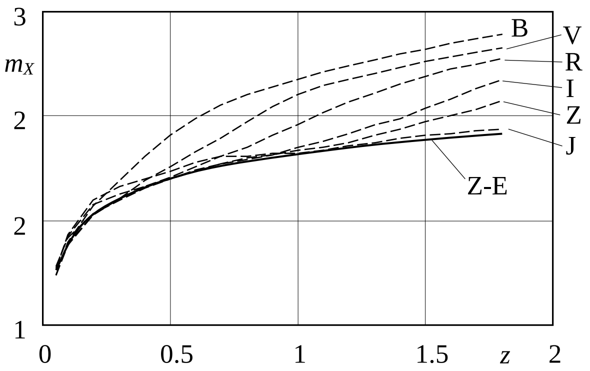
$$K_{i,j}(z) = 2.5\log(1+z) + 2.5\log\left\{ \frac{\int_0^\infty F(\lambda) S_i(\lambda)\, d\lambda}{\int_0^\infty F\left(\lambda/(1+z)\right) S_j\, d\lambda} \frac{\int_0^\infty Z(\lambda) S_j(\lambda)\, d\lambda}{\int_0^\infty Z(\lambda) S_i(\lambda)\, d\lambda} \right\} \qquad (3.6:3)$$



(a) λ_T = 350 nm (λ_W = 440 nm, T = 8300 °K)



(b) λ_T = 440 nm (λ_W = 560 nm, T = 6600 °K)



(c) Tonry et al., [12] Table 7.

FIG. 3.6-3 (a,b). Magnitudes observed in filters BVRIZJ at distance $D = R_4 \cdot z/(1+z)$ from blackbody sources at temperatures 8300 °K and 6600 °K, respectively. (c). Plot of the peak magnitudes of normal Sn Ia observed in BVRIYJ filters as presented by Tonry *et al*. [13] in Table 7. The transmission functions of the filters used by Tonry et al. are slightly different from the transmission functions used in calculations for (a) and (b). See Appendix B for the definitions of λ_T and λ_W. The zero-energy prediction (3.5:10) for the magnitudes in optimally chosen filters is shown by the (Z-E) curve in each figure.

To make equation (3.5:10) consistent with the *K*-corrected magnitudes, it must be written as

$$m_{(K)} = M + 5\log\frac{R_4}{D_0} + 5\log z - 2.5\log(1+z) + K \qquad (3.6:4)$$

Applying equation (3.6:3) for an optimal choice of filters matching the central wavelength of the filter to the wavelength of the maximum of redshifted radiation the *K*-correction becomes

$$K_{opt}(z) \approx 5\log(1+z) \qquad (3.6:5)$$

Substitution of (3.6:5) for *K* in equation (3.5:10) gives the zero-energy space prediction for *K*-corrected magnitudes

$$m_{X(opt)} = M + 5\log\frac{R_4}{d_0} + 5\log z + 2.5\log(1+z) \qquad (3.6:6)$$

The prediction for *K*-corrected magnitudes in the standard model, corresponding to equation (3.6:6) for *K*-corrected magnitudes in zero-energy space, is given by equation

$$\begin{aligned} m &= M + 5\log\left(\frac{R_H}{10\ \text{pc}}\right) + 5\log\left(\frac{D_L}{R_H}\right) \\ &= M + 43.2 + 5\log\left[(1+z)\int_0^z \frac{1}{\sqrt{(1+z)^2(1+\Omega_m z) - z(2+z)\Omega_\lambda}}\,dz\right] \end{aligned} \qquad (3.6:7)$$

where $R_H = c/H_0 \approx 14 \cdot 10^9$ l.y. is the Hubble distance, the standard model replacement of R_4 in zero-energy space. D_L in (3.6:7) is the luminosity distance defined as

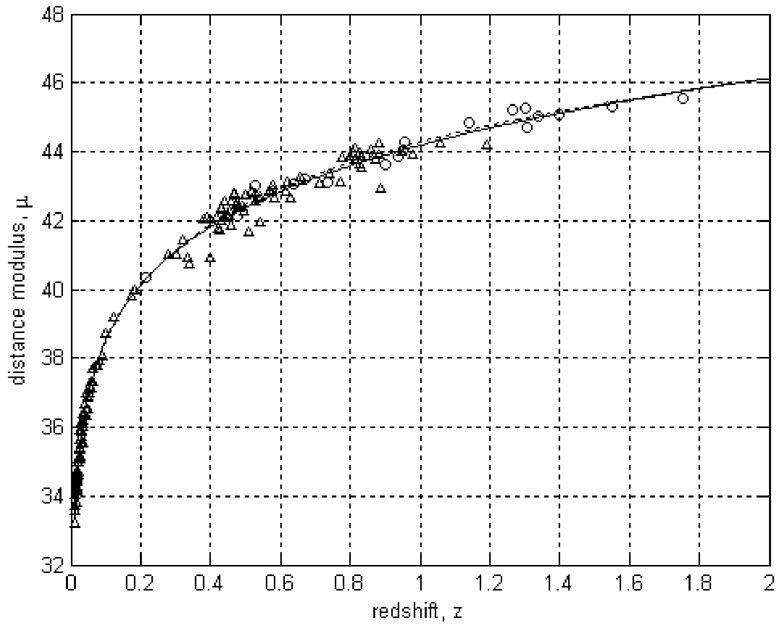


FIG. 3.6-4. Distance modulus $\mu = m - M$, vs. redshift for Riess et al.'s gold dataset and the data from the HST. The triangles represent data obtained via ground-based observations, and the circles represent data obtained by the HST, Riess [15]. The optimum fit for the standard cosmology prediction (3.6:7) is shown by the dashed curve, and the fit for the zero-energy space prediction (3.6:6) is shown, slightly below, by the solid curve, Suntola & Day [16].

$$D_L = D_A \left(1+z\right)^2 = \left(1+z\right)\frac{c}{H_0}\int_0^z \frac{1}{\sqrt{\left(1+z\right)^2\left(1+\Omega_m z\right) - z\left(2+z\right)\Omega_\Lambda}}\,dz \qquad (3.6:8)$$

where D_A is the angular size distance of the standard model given in (3.3:3). Mass density parameters Ω_m and Ω_Λ give the density shares of mass and dark energy in space. For a flat space condition the sum $\Omega_m + \Omega_\Lambda = 1$.

The best fit of equation (3.6:7) to the *K*-corrected magnitudes of Ia supernova observations has been obtained with $\Omega_m = 0.26 \ldots 0.31$ and $\Omega_\Lambda = 0.74\ldots0.69$ [17–23]. Figure 3.6-4 shows a comparison of the prediction given by equation (3.6:7) with $\Omega_m \approx 0.31$, $\Omega_\lambda \approx 0.69\ \Omega$ and $H_0 = 64.3$ used by Riess *et al*. [15] and the zero-energy space prediction for *K*-corrected magnitudes in equation (3.6:6).

In the redshift range $z = 0\ldots2$ the apparent magnitude of equation (3.6:7) coincides accurately with the magnitudes of equation (3.6:6). The *K*-corrections used by Riess *et al*. [15], Table 2, follow the $K(z) = 5{\cdot}\log(1+z)$ prediction of equation (3.6:5), (see Fig. 3.6-5).

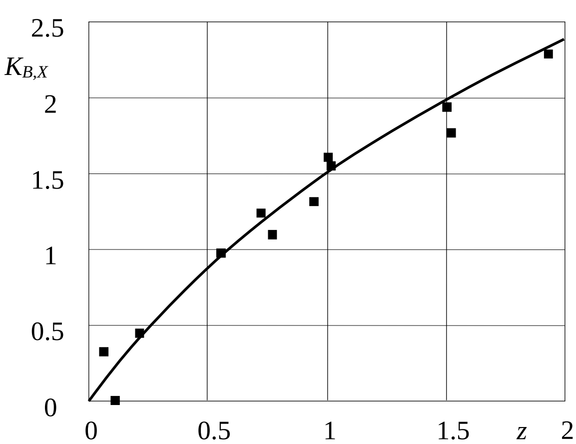


FIG. 3.6-5. Average $K_{B,X}$-corrections (black squares) collected from the $K_{B,X}$ data in Table 2 used by Riess *et al*. [15] for the *K*-corrected distance modulus data shown in Figure 3.6-4. The solid curve gives the theoretical *K*-correction (3.6:5), $K = 5{\cdot}\log(1+z)$, derived for filters matched to redshifted spectra and applied in equation (3.6:6) for the zero-energy space prediction of *K* corrected apparent magnitude.

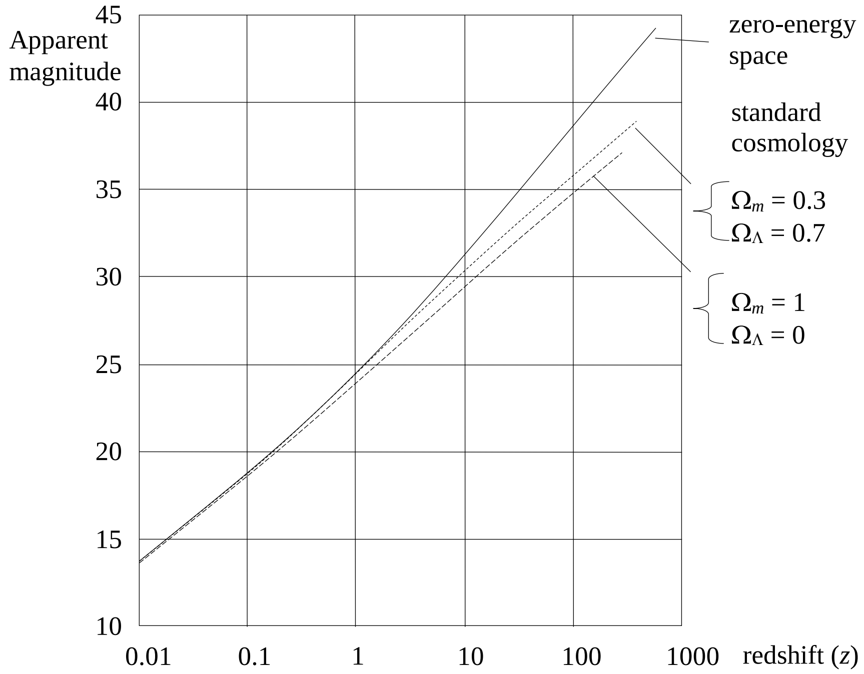


FIG. 3.6-6. Comparison of predictions for the *K*-corrected apparent magnitude of standard sources in the redshift range 0.01...1000 given by the Standard Cosmology Model with Ω_m=0.3/Ω_Λ=0.7 and Ω_m=1/Ω_Λ=0 according to equation (3.6:7), and the zero-energy space given by equation (3.6:6). In each curve the absolute magnitude used is M = –19.5. The Ω_m=0.3/Ω_Λ=0.7 prediction follows the zero-energy prediction closely up to redshift $z \approx 2$, the Ω_m=1/Ω_Λ=0 prediction of the standard model shows remarkable deviation even at smaller redshifts.

At redshifts above $z > 2$ the difference between the two predictions, (3.6:6) and (3.6:7), becomes noticeable and grows up to several magnitude units at $z > 10$ (see Fig. 3.6-6). For comparison, Figure 3.6-6 shows also the standard model prediction for $\Omega_m = 1$ and $\Omega_\lambda = 0$.

3.7 Surface brightness of expanding objects

The Tolman test [12, 24–26] is considered as a critical test for an expanding universe model. In expanding space, according to Tolman's prediction, the observed surface brightness of standard objects decreases by the factor $(1+z)^4$ with the redshift. Two of the four (1+*z*) factors are explained as consequences of the redshift on the radiation received: a decrease in the arrival rate (the number effect) and in the energy of photons (the energy effect), as discussed in Section 3.5. The two additional (1+*z*) factors are explained as an apparent increase in the observed area due to aberration.

In zero-energy space, galaxies and quasars are expanding objects. With reference to equation (3.4:1) the angular area of expanding objects with a present radius r_e is

$$\Omega_D = \left(\frac{r_{e(z)}}{D}\right)^2 = \frac{r_e^2}{(1+z)^2}\frac{(1+z)^2}{R_4^2 z^2} = \frac{r_e^2}{R_4^2}\frac{1}{z^2} \tag{3.7:1}$$

where D is the optical distance of the object. The observed bolometric surface brightness of an object is obtained by dividing the bolometric energy flux by the angular size of equation (3.7:1). When related to the surface brightness of a reference object at distance d_0 ($z_{do} << 1$) the surface brightness of an object at distance D is obtained by applying equation (3.5:9) as

$$\frac{SB_{(D)}}{SB_{(d_0)}} = \frac{F_D}{F_{d_0}}\frac{\Omega_{d_0}}{\Omega_D} = \frac{d_0^2}{R_4^2}\frac{(1+z)}{z^2}\frac{\Omega_{d_0}}{\Omega_D} = \frac{d_0^2}{R_4^2}\frac{(1+z)}{z^2}\frac{r_e^2}{d_0^2}\frac{R_4^2 z^2}{r_e^2} = (1+z) \tag{3.7:2}$$

or

$$SB_{(D)} = SB_{(d_0)}\left(1+z\right) \tag{3.7:3}$$

or related to the K-corrected energy fluxes in a multi-bandpass system with nominal filter wavelengths matched to the redshifted radiation [see Section 3.7] as

$$SB_{K(D)} = SB_{(d_0)}\left(1+z\right)^{-1} \tag{3.7:4}$$

The predictions of equations (3.7:3) and (3.7:4) do not include the effects of possible evolutionary factors.

Lubin and Sandage [27–30] give a thorough review of the theoretical and observational aspects of the Tolman $(1+z)^{-4}$ surface brightness prediction as a test of the FLRW expansion. They conclude that observations of the light curves from supernovas have confirmed the cosmological time dilation [31] as a unique proof of an expanding space. They also interpret the precise Planckian shape of the background radiation as a solid proof of the Tolman surface brightness prediction. However, the observed surface brightnesses of high z objects do not follow the Tolman $(1+z)^{-4}$ prediction without assumptions of remarkable evolution in the luminosity and size of the objects. Equations (3.7:3) and (3.7:4) show the effect the expansion of local systems, like galaxies and quasars, on the observed surface brightness. A detailed analysis of the consistency of predictions (3.7:3) and (3.7:4) with observations of surface brightness is left outside the scope of this paper.

3.8 The effects of the declining velocity of light

As a consequence of the conservation of the zero-energy condition assumed, all velocities in space are related to the velocity of light determined by the expansion in the direction of the 4-radius. Emission of quanta from a supernova explosion occurs at a frequency proportional to the velocity of light at the time of the explosion. A sequence of waves from an explosion is redshifted and accordingly received lengthened in the same ratio as the wavelengths are lengthened, i.e. in direct proportion to $(1+z)$. In the standard model, the lengthening is referred to as cosmological time dilation, in zero-energy space it is a direct consequence of reduced velocity of light at the time the wave sequence is received.

The declining rest energy of matter in zero-energy space makes all atomic processes slow down with the expansion of space; ticking frequencies of atomic clocks and the rate of nuclear decay slow down in direct proportion to the decrease of the velocity of light. The present estimates for the oldest globular clusters, based on constant decay rates observed today, are in the range of 12 to 20 billion years [6].

As given in equation (2.2:6), the age of expanding zero-energy space is $T = (2/3)\cdot R_4/c = (2/3)/H_0$ which means about 9.3 billion years for R_4 = 14 billion light years consistent with the Hubble constant H_0 = 70 [(km/s)/Mpc]. Linear age estimates up to 14 billion years are reduced below the age of zero energy space (see Fig. 3.8-1).

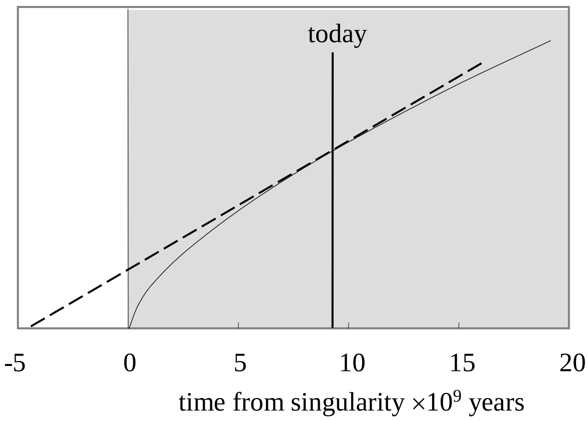


FIG. 3.8-1. Accumulation of nuclear decay products at today's decay rate (dashed line), and at a rate proportional to the velocity of light in zero-energy space (solid curve).

3.9 Microwave Background radiation in zero-energy space

The bolometric energy density of cosmic microwave background radiation, $4.2\cdot 10^{-14}$ [J/m^3], corresponds, with a high accuracy, to the energy density *within a closed blackbody source* at 2.725 °K. (Obs. As indicated by the Stefan-Boltzmann constant, the energy density *within a blackbody source* is, by a factor of 4, higher than the integrated energy density of the flux radiated by the source)

$$E_{bol(T=2.725\ °\mathrm{K})} = E_\nu d\nu = \int_0^\infty \frac{8\pi h}{c^3}\nu_0^3 \frac{(\nu/\nu_0)^3}{\left(e^{\nu/\nu_0}-1\right)} d\nu = 4.2\cdot 10^{-14} \qquad \left[\frac{\mathrm{J}}{\mathrm{m}^3}\right] \qquad (3.10{:}1)$$

where

$$\nu_0 \equiv \frac{kT}{h} = \frac{c}{\lambda_0} \qquad [\mathrm{Hz}] \qquad (3.10{:}2)$$

from which $\nu_0 = 5.6910^{10}$ Hz is obtained for $T = 2.725$ °K.

The rest energy calculated for the total mass in space is $E_{rest} = M_\Sigma \cdot c^2 \approx 2\cdot 10^{70}$ [J] corresponding to energy density $E_{rest}/(2\pi^2 R_4^3) = 4.6\cdot 10^{-10}$ [J] in zero-energy space. Accordingly, the share of the CMB energy density of the total energy density in space is about 10^{-4}. The total mass equivalence, and hence the ratio to the rest energy in space is conserved. The wavelength of radiation is redshifted as

$$z = \frac{R_4 - R_{4(e)}}{R_{4(e)}} = \frac{R_4}{R_{4(e)}} - 1 \qquad (3.10{:}3)$$

where $R_{4(e)}$ is the 4-radius of space at the time of the emission of the CMB. The zero-energy concept does not give a prediction for the value of the 4-radius $R_{4(e)}$ at the emission of the CMB — or exclude the possibility that the CMB is generated continuously by dark matter now at 2.725 °K effective temperature (see Fig. 3.9-1).

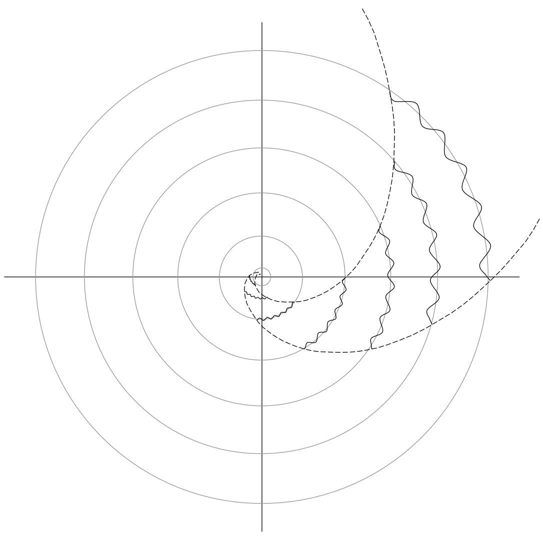


FIG. 3.9-1. The CMB has the characteristics of a closed blackbody source. The number of quanta in radiation in spherically closed zero-energy space is conserved. The wavelength, however, is increased in direct proportion to the expansion of the 4-radius. At present, the energy density of the 2.725 °K background radiation is about $4\cdot 10^{-14}$ [J/m^3] which is about 0.01 % of the energy density of all mass in space.

4. Discussion

Relativity as an implication of the conservation of energy

The gravitational energy due to all mass in zero-energy space can be interpreted as the non-localized, complementary counterpart of the localized rest energy of an energy object. The complementary pair of the energies preserves the zero energy condition in any local energy frame as expressed by equation

$$E_{rest} = c_0 mc = m_0 c_0^2 \prod_{i=0}^{n} \left(1-\delta_i\right) \sqrt{1-\beta_i^2} = -\frac{GM_4 m_0}{R_{4(0)}} \prod_{i=0}^{n} \left(1-\delta_i\right) \sqrt{1-\beta_i^2} \qquad (4:1)$$

which also shows that the rest energy of mass *m* can be equally expressed in terms of the local velocity of light or the gravitational energy due to the entire mass in space.

Relativity in zero-energy space does not rely on the Lorentz transformation, the relativity principle, or the equivalence principle. In zero-energy space all manifestations of relativity are direct consequences of the conservation of the total energy and the zero-energy balance in space. A local state of rest in zero-energy space is determined by the zero-momentum and zero-angular momentum state of the local energy frame studied. The chain of nested energy frames in space relates any local energy state to the state of rest in hypothetical homogeneous space.

The state of rest in a local energy frame serves as the reference for the momentum and kinetic energy of objects moving in the local frame. The zero-energy approach makes a definite distinction between *motion as an expression of momentum and kinetic energy* and *motion as kinematic velocity*, which describes the rate of change in the distance between objects. Velocity as an expression of kinetic energy is related to the state of rest in the frame the kinetic energy is obtained. The kinetic energies of two objects moving at different velocities in a local frame can not be solved from the relative velocity between the objects. *Kinematic velocities*, can be summed up using the Galilean transformation but the resulting relative velocity has very little to do with the kinetic energy of the moving objects.

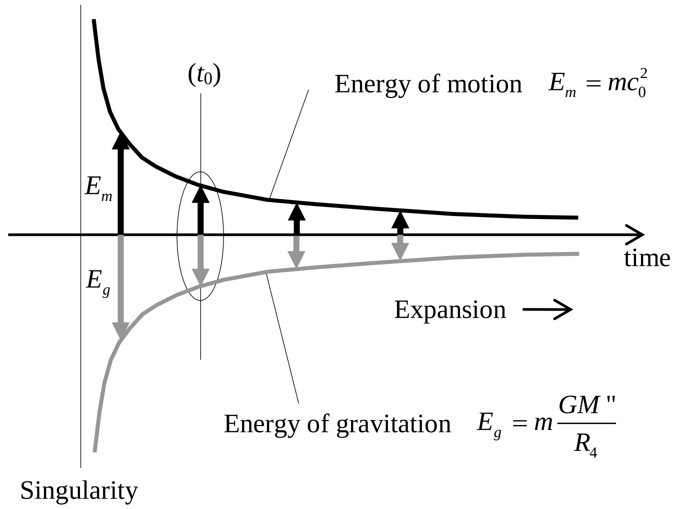
Historically, the basis of zero-energy frames was first time implicitly expressed by Gottfried Leibniz, the great philosopher, mathematician, and physicist contemporary with Isaac Newton. Leibniz introduced the zero-energy principle by stating that *vis viva*, the living force mv^2 (kinetic energy) is obtained against the release of *vis mortua*, the dead force (potential energy) (Leibniz [32]). Indirectly, such an approach defines the state of rest as a property of an energy system where kinetic energy (*vis viva*) is created. This is a major difference from the Galilean – Newtonian – Einsteinean approach of linking the state of rest to an inertial state, a state of zero acceleration. In zero-energy space *energy* is the primary quantity and *force* is a derived quantity; being defined as the negative of the gradient of energy. The linkage of force to acceleration is derived from the zero-energy postulate.

Instead of being derived from field equations like in FLRW space, the local geometry (rather than metrics) of space in Dynamic Universe is described as an equipotential surface in terms of an algebraic, complex presentation of the total energy. Mathematically, it means a major simplification to the field equation based metrics of FLRW space. The zero-energy approach avoids the infinity problem of the field equations at local singularities in space. Local singularities in zero-energy space allow circular orbits down to the critical radius.

The primary conserved quantity in zero-energy space is mass, which can be characterized as the substance for the expression of energy. The zero total energy comes from the sum of the positive energy of motion and the negative energy of gravitation obtained in a contraction – expansion process of spherically closed space, Fig. 4-1.

In atoms the Bohr radius is conserved, which means that the dimensions of all material objects are conserved. As a consequence of the conserved Bohr radius, the wavelengths of characteristic radiation emitted by atoms are unchanged in the course of the expansion.

FIG. 4-1. The dualistic expression of the rest energy of matter. In the expansion of space the rest energy gained in a pre-singularity contraction phase is gradually released back to the gravitational energy. The positive rest energy of a mass object is the localized expression of energy, counterbalanced by the non-localized expression of the negative gravitational energy.



As shown by the analysis of Maxwell's equations for the emission of electromagnetic radiation by an electric dipole, the energy of a quantum is linked to the energy carried by a cycle of radiation. The wavelength of electromagnetic radiation propagating in space increases in direct proportion to the expansion. Conservation of the mass equivalence of a quantum of electromagnetic radiation means that the energy of a quantum is conserved in relation to the rest energy of mass in space. Figure 4-2 illustrates the expanding and non-expanding energy structures in Dynamic Universe.

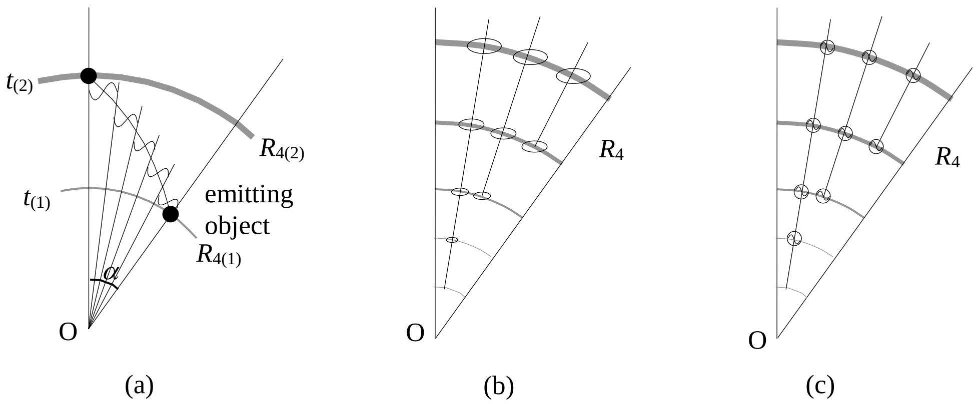


FIG. 4-2. Conservation of dimensions and the mass and the wavelength of radiation: (a) The redshift of radiation results from the lengthening of the wavelength with the expansion of space. The number of quanta (or wavelengths of radiation) on the way from an emitter of constant intensity to an observer at a fixed distance angle from the emitter is constant with time. (b) The dimensions of galaxies and other gravitationally bound systems expand in direct proportion to the expansion of space. (c) Localized objects bound by electromagnetic of nuclear forces conserve their size. The characteristic wavelength emitted by atomic objects is conserved in the course of the expansion of space.

Electromagnetic radiation propagating in space looses power when received with an increased wavelength and consequently at a lowered frequency due to the redshift. The energy of a quantum, linked to the energy carried by of one wavelength of radiation, is conserved throughout the propagation.

Singularity as the hot big bang replacement

Instead of a sudden appearance of mass and energy, singularity of zero-energy space is seen as the turning point of a contraction phase into the ongoing expansion phase. With regard to the wave nature of mass we may assume a quantum limit to the 4-radius at passing the singularity. Such a limit could work as a measurement rod to structures maintaining their dimensions in expanding space. The basic form of matter in hypothetical homogeneous space is considered to have non-structured homogeneous radiation-like appearance with momentum in the direction of the 4-radius.

At infinity in the past, like at infinity in the future, the 4-radius of space is infinite. Mass as the substance for the expression of energy exists, but as it is not energized it is not detectable. The energy of motion built up in the primary energy build-up is gained from the structural energy, the energy of gravitation. Space loses size and gains motion (see Fig. 4-3).

At infinity in the future, all motion gained from gravity in the contraction will have been returned back to the gravitational energy of the structure in the expansion. Mass will no longer be observable because the energetic excitation of matter will have vanished along with the cessation of motion. The energy of gravitation will also become zero owing to the infinite distances. *The cycle of observable physical existence begins in emptiness and ends in emptiness where the mass does not express itself as energized matter.*

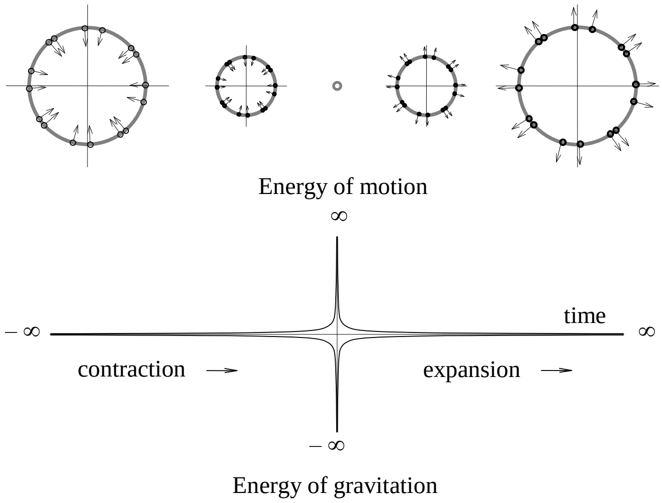


FIG. 4-3. The primary energy build-up and release in spherical space. In the contraction phase, the velocity of the imaginary motion increases due to the energy gained from loss of gravitation. In the expansion phase, the velocity of the imaginary motion gradually decreases, while the energy of motion gained in contraction is returned to gravity.

Cosmological observations

The analysis of distances and magnitudes in zero-energy space means major simplification in comparison to the corresponding analyses in FLRW space. A fundamental difference between the FLRW space and the zero-energy space is that, unlike in FLRW space, gravitationally bound local systems in zero-energy space are subject to expansion in direct proportion to the expansion of the 4-radius. As a consequence, the predictions for the observed angular size and surface brightness of galactic objects are different from those in FLRW space. The linkage of a quantum of radiation to the energy carried by one cycle of radiation cancels the "energy effect", included in addition to the "number effect" as a second (1+z) dilution factor to the power density of redshifted radiation in the standard model.

Predictions for cosmological observables in zero-energy space are based on very few assumptions — closing the three-dimensional space in spherical symmetry and maintaining a zero-energy balance by accepting motion of space in the direction of the 4-radius of the structure. The zero-energy balance in space defines the mass density in space, links the velocity of light to the expansion of space — and cancels the need for a cosmological constant.

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