On the Planck Scale and Structures of Matter

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Nonlinearity

Nonlinearity is common property of natural phenomena.

The best known example is the *1/r* gravitational and Coulomb potentials

Energy gradients, or forces, are even more nonlinear.

Period doubling is a universal property of *nonlinear dynamical systems* [1].

Even a simple pendulum may show period doubling.

[1] P. Cvitanović and M. Høgh Jensen, ed's, "Chaos and Universality", Nordita preprint selection, November 1981

Period doubling

What is it?

If the period of the fundamental oscillation is τ_o, then period doubling generates a sequence of periods:

$$\tau_N = 2^N \tau_o$$

where N is a positive integer.

Period doubling is the same as frequency halving, because

$$f = \frac{1}{\tau}$$

and frequency spectrum $f_o,\ f_o/2,\ f_o/4,\ f_o/8$ etc. is borne.

The Planck scale

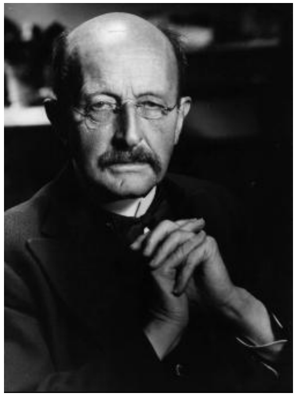
Although period doubling is a common property of nonlinear dynamical systems its possible occurrence at the **Planck scale** has been seldom studied.

The Planck scale is determined by natural constants, and its connection to the observable world is not well known.

The Planck scale is determined by

- the Planck constant h
- the gravitational constant G
- the speed of light c
- the permittivity of free space ε_o

Motivation of this study



Max Planck
1858–1947
These [the Planck units] necessarily retain their meaning for all times and for all civilizations, even extraterrestrial and non-human ones, and can therefore be designated as "natural units"...
1899

To find out whether period doubling at the Planck scale might be responsible for the invariant properties and structures of matter.

The Planck units:

Mass	$m_o \approx 5.46 \cdot 10^{-08}$	kg	$(m_o^2 = hc/G)$
Energy	$E_o \approx 3.06 \cdot 10^{22}$	MeV	$(E_o = m_o c^2)$
Time (period)	$\tau_o \approx 1.35 \cdot 10^{-43}$	s	$(\tau_o = h/E_o)$
Frequency	$f_o \approx 7.41 \cdot 10^{42}$	s	$(f_o = 1/\tau_o)$
Charge	$q_o \approx 4.70 \cdot 10^{-18}$	As	$(q_o^2 = hc \cdot 4\pi\varepsilon_o)$
Length	$l_o \approx 4.05 \cdot 10^{-35}$	m	$(l_o = c\,\tau_o)$

This is (part of) the "Planck scale".

[1] Note: units are defined by h (not h–bar)

We now assume that

1. A nonlinear dynamical system at the Planck scale generates ***sub-harmonic frequencies*** by ***multidimensional*** period doubling
2. The periods, or sub-harmonic frequencies, can be related to other physical quantities by known relations
   - Planck relation $E=hf=h/\tau$ (τ = period)
   - length or distance $l=c\tau$
   - magnetic moment $\mu=iA$ ($i=e/\tau$, $A=l^2$)

Definition of a system: A naturally occurring group of objects or phenomena

- e.g. the internal structures of elementary particles or planets in the Solar system

Energy levels

Let us start with energy

According to the Planck relation $E = hf$.

Period doubling generates a sequence of energy levels

$$E_N = hf_N = h(2^{-N} f_o) = 2^{-N} E_o$$

where E_o is the unit (=Planck) energy.

The subharmonic frequencies create a discrete spectrum of *decreasing* energies in a natural way contrary to harmonic frequencies, which generate higher energies (than the Planck energy).

Stability condition

In principle the total number of doublings can be any integer, but only a few structures of matter are stable (or long lived).

- electron
- proton
- planetary system

It is known in chaos theory that for *stable periods* the number of doublings is $N=2^m$

$$\tau_N = 2^N \tau_o = 2^{2^m} \tau_o$$

where m is a positive integer.

For stable (or long lived) periods ***N=1, 2, 4, 8, 16, 32, 64, 128 …***

Generalization of Period Doubling (1)

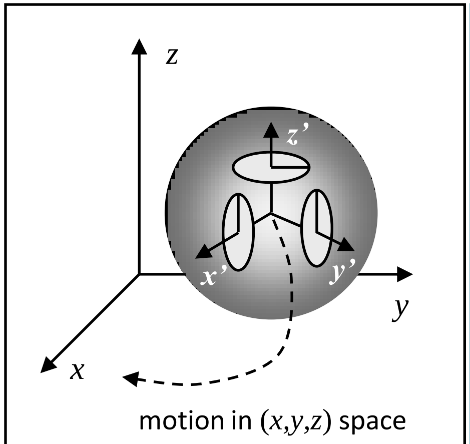


Fig. 1. System (the ”sphere”) with internal degrees of freedom. (x',y',z') are the axes of internal rotation. The system can freely move in space.

Something like three “spins”

Real structures of matter are three dimensional. We therefore generalize period doubling into multiple dimensions.

We define a **system** by *mutually independent internal degrees of freedom*, which can be described by *periods* of rotation.

Such a system is shown in fig. 1, where the “sphere” is the system under consideration.

(x',y',z') are the axes of internal rotation and the system itself is free to move in the (x,y,z)-space.

Generalization of Period Doubling (2)

For a three dimensional system we define three degrees of freedom, i.e. periods τ_i, τ_j and τ_k , which double as usual:

$$\tau_i = 2^i \tau_o \qquad \tau_j = 2^j \tau_o \qquad \tau_k = 2^k \tau_o$$

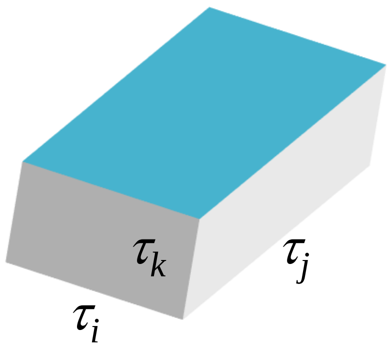
where i, j, k are positive integers. The volume of the **τ-space** parallelepiped is the product of the individual periods

$$V_{ijk}(\tau) = 2^i \tau_o \cdot 2^j \tau_o \cdot 2^k \tau_o = 2^{i+j+k} \tau_o{}^3 \qquad (1)$$

where $i+j+k$ is the **total** number of **volumetric** doublings.

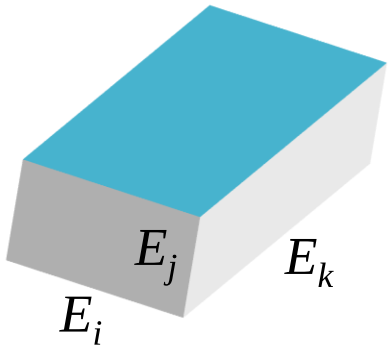
In τ-space (1) defines both ***shape*** (distribution) and ***size*** (magnitude) for the system under consideration.

Generalization of Period Doubling (3)



τ-space, ***shape*** and ***size***

$$E = hf = \frac{h}{\tau}$$



E-space, ***shape*** and ***size***

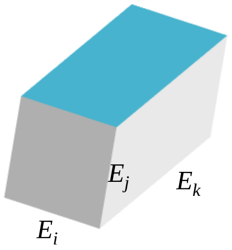
- internal energy distribution

τ-space defines E-space!

Perceived internal energy

By the Planck relation energy (cubed) becomes

$$E_{ijk}{}^{3}(\tau) = \frac{h^3}{V_{ijk}(\tau)} = \underbrace{2^{-i} E_o}_{E_i} \cdot 2^{-j} E_o \cdot 2^{-k} E_o = 2^{-(i+j+k)} E_o{}^{3}$$



Energy cubed retains shape and size. But we do not perceive (=measure) energy cubed!

The *perceived energy* (in Joules) is cube root of $E_{ijk}{}^{3}$

$$E_{ijk} = 2^{-\frac{i+j+k}{3}} E_o \qquad \text{(geometric mean of the edge lengths!)} \qquad (2)$$

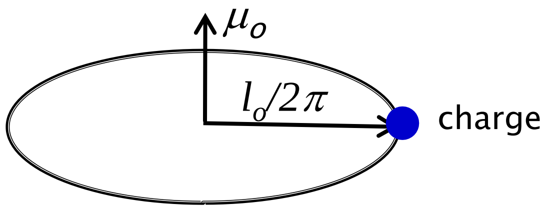
where $(i+j+k)/3$ is the *perceived* number of doublings.

Planck scale magnetic moment μ_o

Conventional definition: $\mu = iA$ (current times loop area)

The unit magnetic moment can be modeled by a current loop, whose circumference is (in the Bohr sense) one Planck length l_o and current $i_o = e/\tau_o$.

Fig.2.



$$\mu_o = i_o \cdot A_o = \frac{e}{\tau_o} \cdot \pi\left(\frac{l_o}{2\pi}\right)^2 = \frac{ec^2}{4\pi}\tau_o \qquad (3)$$

Numeric value is: $\mu_o = 1.5485 \cdot 10^{-46}$ Am^2 (for the electron $\mu_e = 9.28 \cdot 10^{-24}$ Am^2)

Experimental

Let's see whether

- the rest masses of the electron and proton (the only stable particles)
- the elementary electric charge
- the fine structure constant
- the magnetic moment of the electron
- the Bode-Titius rule, i.e. the structure of the Solar system

could be consequences of period doubling taking place at the Planck scale.

More examples in: Ari Lehto, "On the Planck scale and properties of matter", Nonlin. Dyn, Volume 55, Number 3 / February, 2009

Rest Energy

Electron-positron pair

An electron-positron (*ep*) pair can be created in the *pair production* process by a gamma quantum. The rest energy of the pair is $E=E_{ijk}=1.022$ MeV.

Perceived energy by Eq. (2) $$E_{ijk} = 2^{-\frac{i+j+k}{3}} E_o$$

yields: $$E_{ep} = 1.021 MeV = 2^{-\frac{32+64+128}{3}} E_o$$

We find that the internal unit period (=Planck time) has doubled *32*, *64* and *128* times in the three degrees of freedom . The stability condition is fulfilled, too.

Magnetic Moment

Electron-positron pair

The magnetic moment of an electron can be modeled by a current loop as shown in fig. 3.

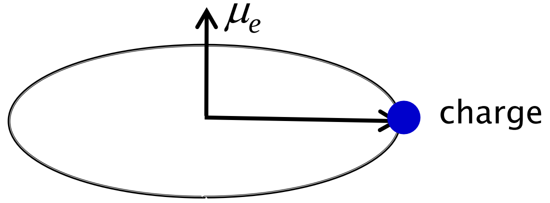


Fig. 3.

According to Eq. (3) magnetic moment is directly proportional to period. Therefore the magnetic moment of an electron is *twice* that of an electron-positron pair [1], or $\mu_{ijk}=\mu_e=2\mu_{ep}$. We find that:

$$\mu_e = 9.286\cdot 10^{-24}\,Am^2 = 2\cdot \underbrace{2^{\frac{32+64+128}{3}}\,\mu_o}_{\mu_{ep\text{-}pair}}$$

The stability condition $i=32$, $j=64$, $k=128$ is fulfilled, too.

[1] For half energy period doubles. Measured $|\mu_e| = 9.285.10^{-24}$ Am^2

Elementary Electric Charge

Charges squared are related to *energy* and thus to $E=h/\tau$. We shall therefore calculate the perceived number of doublings for the ratio of the Planck charge squared to the elementary electric charge squared:

$$\frac{q_o^{\ 2}}{e^2} = 2^{\frac{39}{4}} = 2^{\frac{i+j+k+l}{4}} = 2^{\frac{1+2+4+32}{4}}$$

Stability condition $i=1$, $j=2$, $k=4$ and $l=32$ is fulfilled.

Connection to *ep*-pair is via the E-space 2^{32}-edge.

Note: Perceived fourth root (39/4) means *four degrees of freedom* (earlier we had three for mass-energy!)

Elementary Electric Charge

The numeric value of the elementary electric charge can be calculated from the previous equation:

$$\frac{q_o^{\,2}}{e^2} = 2^{\frac{i+j+k+l}{4}} = 2^{\frac{39}{4}}$$

e^2 becomes: $$e^2 = 2^{-\frac{39}{4}} \cdot q_o^{\,2}$$ ($q_o^2 = 4\pi\varepsilon_o hc$)

and we obtain: $e = \; 1.60213 \cdot 10^{-19}$ As,

which differs from the recommended value by *0.003% (30 ppm)*.

Note: ± means polarity (attraction-repulsion), which mass-energy cube root does not give (attraction only).

The fine structure constant

Definition

The fine structure constant *alpha* is defined: $\alpha = \frac{e^2}{2\varepsilon_o hc}$

Multiplying alpha by $2\pi/2\pi$ yields

$$\alpha = 2\pi \frac{e^2}{4\pi\varepsilon_o hc} = 2\pi \frac{e^2}{q_o^{\;2}} = 2\pi 2^{-\frac{39}{4}}$$

and

$$\alpha^{-1} = \frac{1}{2\pi} 2^{39/4} = 137.045$$

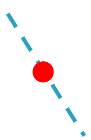
Geometric factor!

The NIST value is $1/\alpha = 137.036$ (0.007% difference)

The Solar system

Newton

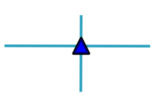
$$v = \sqrt{\frac{GM_{Sun}}{r}}$$



Grid:

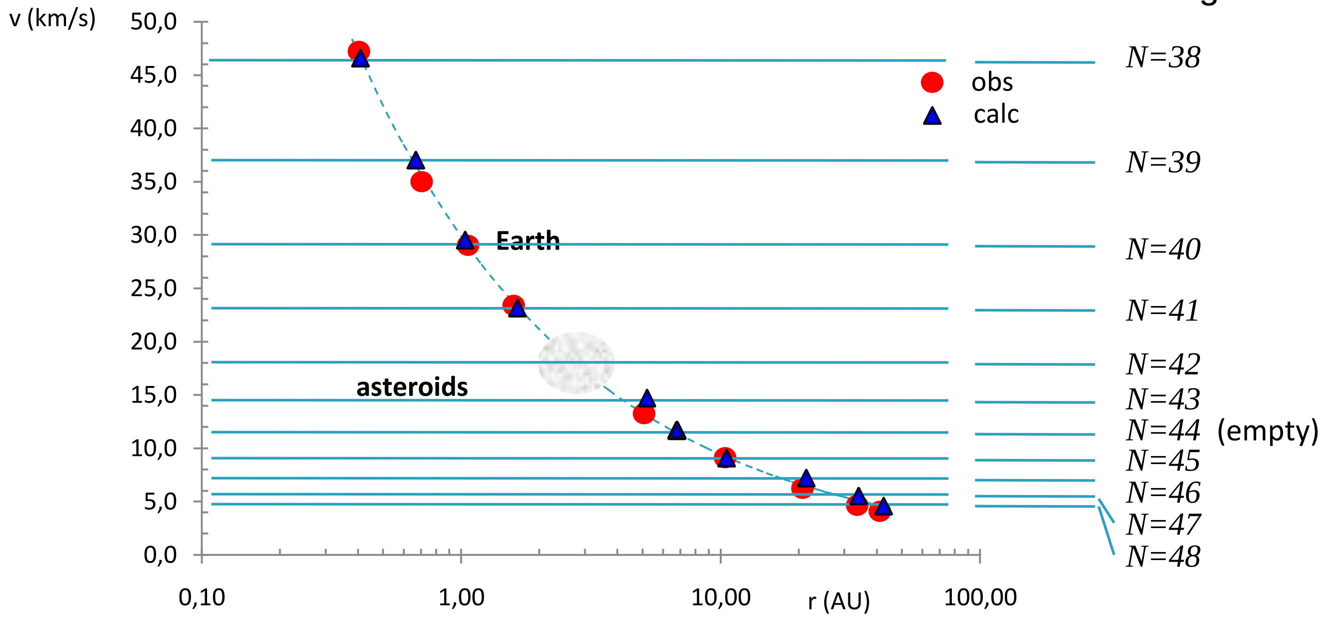
$$r_M = 2^{\frac{M}{3}} l_o$$

$$v_N = 2^{-\frac{N}{3}} c$$



Orbital velocity at semimajor axis vs. semimajor axis

Velocity by consequent doubling

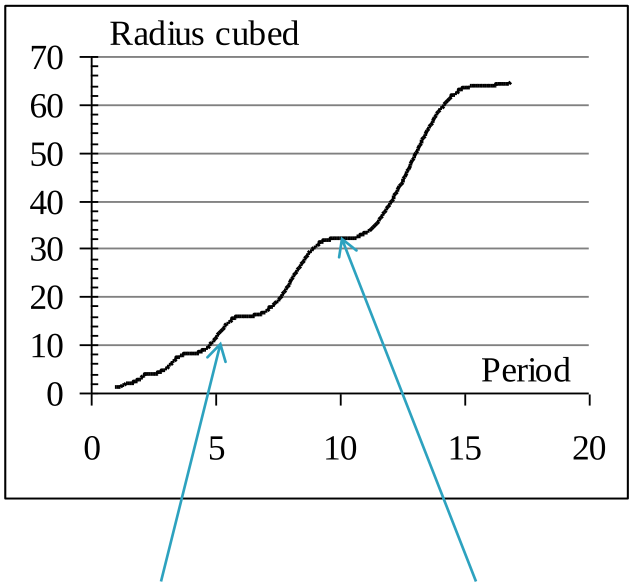


Triangles are at grid points.

At the formation of the Solar system matter accumulated in orbits determined by period doubling. Self-organization. Vertical lines are omitted for clarity.

1/r potential and period doubling

Behaviour of solution to (3).



Transition region

Doubled volume 1, 2, 4, 8, 16, 32, 64 …

Starting from the *1/r* potential and Kepler's third law ($r^3 = const \cdot \tau^2$) it is possible to derive a second order differential equation in (**r, τ**)-space

$$\frac{d^2 r}{d\tau^2} = -\frac{a}{\tau^2} r \qquad (3)$$

which looks like a harmonic oscillator (in τ-space !) except that the spring constant depends on τ, which makes the amplitude grow and oscillation slow down with growing period.

If $a = 46.47$, r^3 doubles, and with

$a = 82.40$, r^4 doubles

Summary and logic

1. gravitational and Coulomb potentials are $1/r$ - nonlinear
2. period doubling is a common property of nonlinear dynamical systems
3. therefore period is used as an internal degree of freedom of a system
4. period doubling is generalized into 3- and 4-degrees of freedom
5. there is a sub-class of stable periods (leading to stable systems)
6. it is assumed that period doubling takes place at the Planck scale
7. therefore units are defined by natural constants (Planck units)
8. calculated values are consistent with observation
9. period doubling seems to be applicable to a wide range of stationary or long lived properties and structures of matter

Thank you!